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REPORT OF ASCE-ACI JOINT COMMITTEE ON ULTIMATE STRENGTH DESIGN

STRUCTURAL DIVISION

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REPORT OF ASCE[1]–ACI JOINT COMMITTEE ON ULTIMATE STRENGTH DESIGN

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Helpful suggestions have been received from Prof. I. M. Viest and others.

The assistance and very substantial contribution to the basic studies and final preparation of the report by A. L. Parme is greatly appreciated by the Committee.

Historical Background

A pronounced interest in the ultimate strength of structural members dates back only one or two decades, but its origin may be found far back in the records of engineering endeavor, further back than the concepts of elasticity and working stresses. The origin of systematic thought regarding flexure of beams, Galilei's work of 1638, was exclusively devoted to ultimate strength. Hooke's Law was formulated 40 years later, and over 180 years elapsed before the fundamental theorems of the theory of elasticity were developed by Navier in 1821.

Several early studies of reinforced concrete members, such as Thullie's flexural theory of 1897 and Ritter's introduction of the parabolic distribution of concrete stresses in 1899, were ultimate strength theories (Fig. 1). The straightline theory of Coignet and Tedesco became generally accepted about 1900. Two primary reasons were given for such acceptance. The straightline theory was mathematically simple, and the resulting safety factors with respect to ultimate loads observed in tests, were sufficiently controlled to satisfy the requirements of that time.

Ultimate strength design is not new even in terms of American specifications. A committee report of the National Association of Cement Users suggested a reinforced concrete code in 1908 with the following design basis: "The design shall be based on the assumption of a load four times as great as

1. Subcommittee of Committee on Masonry and Reinforced Concrete, R. F. Blanks, Chairman, Raymond Archibald, Leo H. Corning, A. E. Cummings,[2] Raymond E. Davis, L. C. Maugh, Raymond C. Reese and Charles S. Whitney.
2. Deceased July 1955.

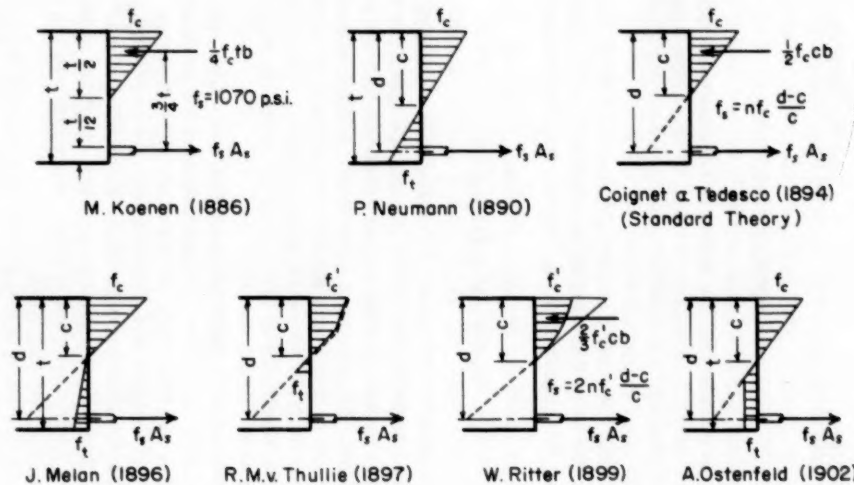

Fig. 1. Early Assumptions in Flexural Analysis (From University of Illinois Engineering Experiment Station Bulletin 399).

the total working load producing a stress in the steel equal to the elastic limit and a stress in the concrete equal to 2000 psi."

An American ultimate strength design code was never adopted, however, since the first Joint Committee on Standard Specifications for Concrete and Reinforced Concrete introduced the concepts of working loads and working stresses, thereby establishing the straightline theory in 1909. Thus, conditions at working loads were emphasized in design, with relatively little attention being devoted to controlling the safety of designs.

In the early years of our century a very rapid development took place in the use of reinforced concrete as a construction material. Many scientific studies were made to aid this development, but few new fundamental concepts were evolved.

The straightline theory became so widely used that there was a tendency to overlook the approximations involved in its assumptions, and applications beyond the range of validity of the theory resulted. For instance, when beams were designed with an allowable concrete compressive stress, f_c, equal to 0.325 times the cylinder strength, f'_c, it was at times believed that the safety factor against a compression failure was near three. In the 1920's Lyse, Slater, and Zipprodt pointed out by beam tests that the safety factor in the case mentioned usually exceeds the ratio f'_c/f_c, thus re-emphasizing the actual inelastic behavior of concrete.

Another important development was initiated by McMillan's study of column test data in 1921, which showed that building columns under load may develop steel stresses, due to creep, considerably higher than those predicted by the straightline theory. This led to the ACI Column Investigation in the 1930's which was carried out by Lyse, Slater, and Richart. Rational equations for the ultimate strength of reinforced concrete columns were established, on the basis of which a major revision of column design procedures in the ACI Code took place.

In 1931 Emperger wrote a critical study of the modular ratio, allowable stresses, and the straightline theory as used in reinforced concrete design. This paper initiated intense theoretical and experimental studies of the ultimate strength of beams, which soon spread over the world. In this country, Jensen and Whitney have made the most important contributions.

In recent years, the literature on ultimate strength design, especially foreign, has become very extensive. In England, impetus was given to the subject by Rhydwyn Harding Evans' paper "The Plastic Theories for the Ultimate Strength of Reinforced Concrete Beams" in 1944. In 1951 a Doctoral Thesis (Deutscher Ausschuss für Stahlbeton, Heft 103) was devoted almost entirely to a review and a comparison of the various theories. A less extensive review was included in Bulletin 399 of the University of Illinois Engineering Experiment Station, also published in 1951.

Some of the theories that have been presented are reviewed in Fig. 2. It is difficult to determine the stress-strain relation of concrete in flexural compression by direct experimental means since measurement of stresses is almost impossible, although strains may be easily observed. The stress distributions shown in Fig. 2 therefore vary somewhat. In terms of predicted ultimate strength of reinforced concrete members, however, if the proper empirical parameters are used there is a relatively small difference between the various theories.

After 1950, a consolidation of knowledge has been carried out, and new important test data have been published. Our knowledge of the entire field of reinforced concrete design has advanced so far that a transition to ultimate

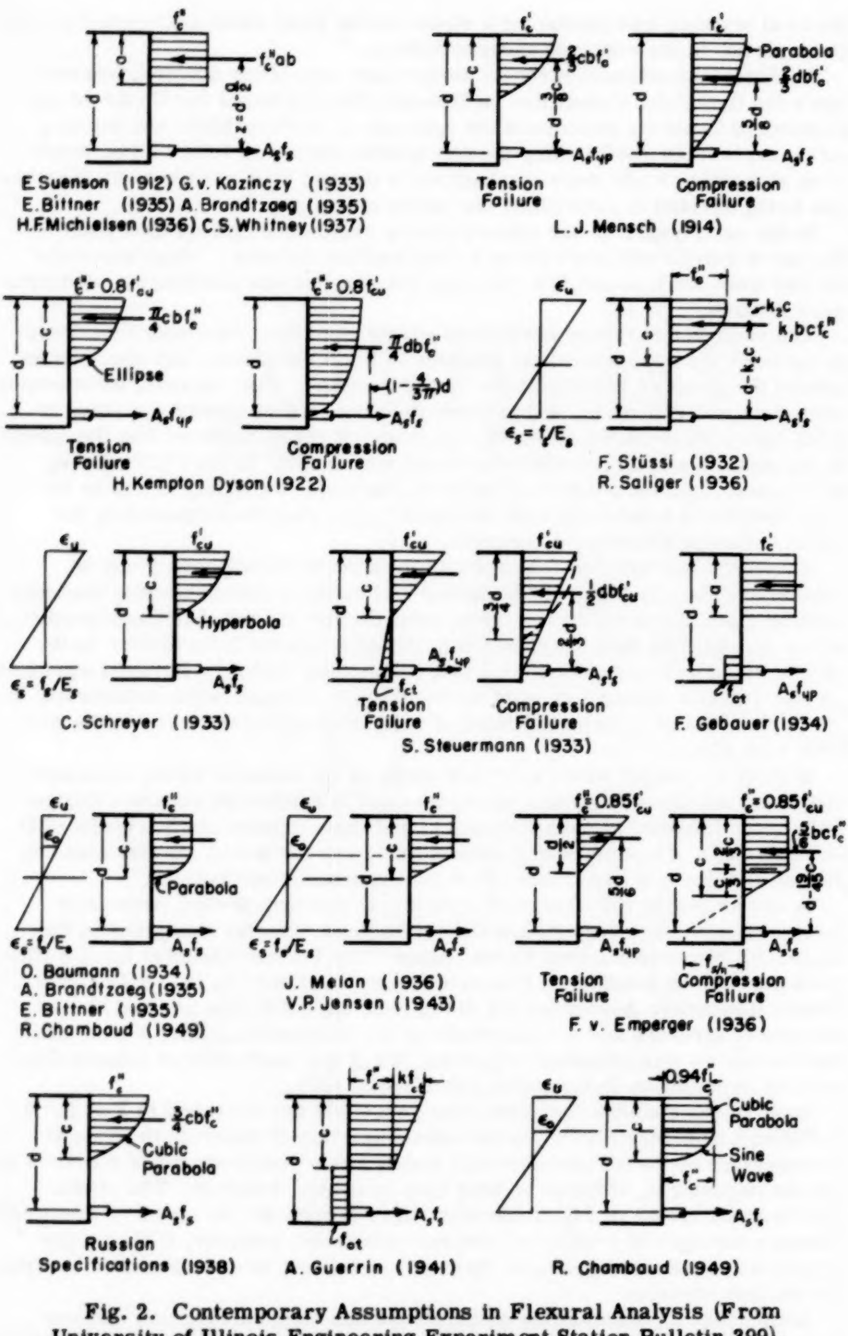

Fig. 2. Contemporary Assumptions in Flexural Analysis (From University of Illinois Engineering Experiment Station Bulletin 399).

strength design seems necessary in order to continue progress. Ultimate strength design has been adopted in Russia, Brazil, and several countries in Europe. Ultimate strength design procedure is being considered in several other countries.

Recommendations for Design

Introduction:

This report presents recommendations and formulas for design of reinforced concrete structures by ultimate strength theories together with basic supporting and explanatory data. The term "ultimate strength design" indicates a method of design based on the ultimate strength of a reinforced concrete cross-section in either simple bending, combined bending and axial load, shear or bond on the basis of inelastic action. The report is confined to design of sections. It does not deal with the evaluation of the external moments and forces that exist in structures.

The Committee recognizes that in indeterminate structures, as ultimate load is approached, there is a readjustment in the relative magnitude of bending moments at various sections due to a nonlinear relationship between load and moment with the moment in the more highly stressed sections increasing at a lower rate than in the sections less highly stressed. This inelastic behavior, commonly referred to as redistribution of moments in limit design, is important, but this report has been confined to design of sections and consideration of their ultimate strength. Studies of the Committee show that the ultimate load capacity of a reinforced concrete section can be predicted with an accuracy within design requirements. This has been verified by comparisons of results obtained by theories and tests, some of which are given in Appendix A.

It is assumed that external moments and forces acting on a structure will be determined on the basis of the theory of elastic displacements. On the basis of this assumption stresses will remain within the elastic limits under service loads when proper load factors are used. For simple beams, the ultimate capacity equals the computed capacity. For indeterminate structures, the maximum moments at various sections are due to different load arrangements. Therefore, the maximum load capacity of a structure may be considerably greater than that indicated by the capacity at one section because of redistribution.

Reasons for Ultimate Strength Design:

The advantages resulting from design of structures for ultimate strength may be summarized as follows:

(a) As ultimate load is approached stress and strain are not proportional. Therefore, the elastic or straightline theory does not give a reliable prediction of the ultimate strength of a section. Under some circumstances, the ultimate strength may be more than 50 per cent greater than that computed by the straightline theory. It follows that the actual factor of safety cannot be determined by the straightline theory. This deficiency is eliminated by ultimate strength design.

(b) Dead load is a determinate quantity that generally remains unchanged during the life of a structure, but actual live loads are less predictable quantities beyond the designer's control. Therefore, it is unreasonable to apply the same load factors to dead and live loads. Ultimate strength design

conveniently permits the use of different factors which results in a more uniformly consistent factor of safety for live load.

(c) Conventional column design is a modified ultimate strength procedure whereas the straightline theory is used for design for simple flexure. It is unavoidable, therefore, that various inconsistencies occur in design of sections subject to both axial load and bending. Designing all types of members on the basis of ultimate strength results in consistency in the design procedures.

(d) A better evaluation of the critical moment thrust ratio for members subject to combined bending and axial load is obtained by ultimate strength design procedure. In many structures, as arches and multiple story frames, the thrust may be due largely to dead load while moment is created primarily by live load. By the use of different overload factors for each type of load, the ultimate strength procedure permits consideration of an increase in live load and thereby moment without a corresponding increase in the dead load or thrust. The combination resulting from this consideration may be more critical than that produced by the same increase in both dead load and live load.

(e) For prestressed concrete it is necessary that design recommendations include investigation of ultimate strength to determine the factor of safety since at high loads, stresses do not vary linearly. The straightline theory is therefore not applicable so the ultimate strength theory must be used.

General Requirements:

(a) The "Building Code Requirements for Reinforced Concrete" by the American Concrete Institute, apply to the design of members by ultimate strength theory except where otherwise provided in this report.

(b) Analysis of indeterminate structures such as continuous bridge girders and arches should be based on the theory of elastic displacements. For buildings of usual types of construction, spans and story heights approximate methods such as the use of coefficients recommended in the ACI Building Code are acceptable for determination of moments, shears and thrusts.

(c) Bending moments in compression members should be taken into account in the calculation of their strength.

(d) In structures such as arches the effect of shortening of the arch axis, temperature, shrinkage and secondary moments due to deflection should be considered.

(e) Attention should be given to the deflection of members, including the effect of creep, especially whenever the net ratio of reinforcement which is defined as $(p-p')$ or (p_w-p_f)* in any section of a flexural member exceeds $0.18f'_c/f_y$.

(f) Controlled concrete should be used and should meet the following requirements. The quality of concrete should be such that not more than 1 test in 10 should have an average strength less than the strength assumed in the design and the average of any three consecutive tests should not be less than the assumed design strength.

Not less than one test should be made for each 250 cu. yd. of concrete, for each day's work, for each class of concrete, and for each change of supplier or source of material. Each test should consist of not less than 3 standard cylinders made, cured and tested in accordance with "Method of Making and Curing Concrete Compression and Flexure Test Specimens in the Field" (ASTM C31) and "Method of Testing for Compressive Strength of Moulded Concrete Cylinders" (ASTM C39).

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* For definition of symbols see pages 8, 10 and 11.

Where there is question as to the quality of the concrete in the structure, the engineer (architect) may require tests in accordance with "Methods of Securing, Preparing and Testing Specimens from Hardened Concrete for Compressive and Flexural Strengths" (ASTM C42) or order load tests of that portion of the structure where the questionable concrete has been placed.

Assumptions:

The design of reinforced concrete members is based on the following assumptions:

1. Plane sections normal to the axis remain plane after bending.
2. Tensile strength in concrete is neglected in sections subject to bending.
3. At ultimate strength stresses and strains are not proportional and the distribution of compressive stresses in a section subject to bending is nonlinear. The diagram of compressive concrete stress distribution may be assumed a rectangle, trapezoid, parabola or any other shape which results in ultimate strength in reasonable agreement with tests.
4. Maximum fiber stress in concrete does not exceed $0.85f'_c$.

Load Factors:

The following terms are used in the load factor equations:

U = ultimate strength of section.
B = effect of basic load consisting of dead load plus volume change due to plastic and elastic actions, shrinkage and temperature.
L = effect of live load plus impact.
W = effect of wind load.
E = effect of earthquake forces.
K = load factor equal to 2 for columns and members subjected to combined bending and axial load, and equal to 1.8 for beams and girders subjected to bending only.

Members should be proportioned so that: (1) They should be capable of carrying without failure the critical load combination given below thereby insuring an ample factor of safety against an increase in live load beyond that assumed in design; (2) The strains under working loads should not be so large as to cause excessive cracking. These criteria are satisfied by the following formulas:

For those structures in which, due to location or proportions, the effects of wind and earthquake loading can properly be neglected:

$$U = 1.2B + 2.4L \qquad \text{(I)}$$
$$U = K(B + L) \qquad \text{(II)}$$

For those structures in which wind loading should be considered:

$$U = 1.2B + 2.4L + 0.6W \qquad \text{(Ia)}$$
$$U = 1.2B + 0.6L + 2.4W \qquad \text{(Ib)}$$
$$U = K\left(B + L + \frac{W}{2}\right) \qquad \text{(IIa)}$$
$$U = K\left(B + \frac{L}{2} + W\right) \qquad \text{(IIb)}$$

For those structures in which earthquake loading should be considered, substitute E for W in the preceding equations. In case there is doubt as to the importance of wind or earthquake loading it can be tested by making a trial calculation using these equations.

Rectangular Beams with Tensile Reinforcement Only:

As the ultimate capacity of a section to resist moment and thrust is approached, the distribution of compressive stresses assumes a shape such as that shown in Fig. 3. However, any idealized shape such as rectangular, trapezoidal or parabolic, may be used in design provided that good agreement is obtained with ultimate strengths measured in comprehensive tests such as those listed in Appendix A.

The ultimate capacity of an underreinforced section is approached when the tensile steel begins to yield. The steel is then assumed to elongate plastically at its yield point stress thereby reducing the concrete area in compression until the concrete crushes at its ultimate strain. Because of the initial yielding of the steel, the ultimate strength so obtained is considered to be controlled by tension.

The equations for the ultimate strength of a section are based on the two equations of equilibrium: (1) the sum of internal stresses equals the external axial force, and (2) the moment of the internal stresses equals the external moment. For rectangular underreinforced beams, in which the strength is limited by tension in the steel, the ultimate resisting moment is given by the formula:

$$M_u = A_s f_y d \left(1 - \frac{k_2}{k_1} \times \frac{p f_y}{0.85 f'_c}\right) \quad (1)$$

in which

A_s = area of reinforcement.
f_y = yield point stress of steel or 60,000 psi. The smaller of the two values should be used.
d = distance from extreme compressive fiber to centroid of tensile reinforcement.
k_2 = ratio of distance between extreme fiber and resultant of compressive stresses to distance between extreme fiber and the neutral axis, or ratio of distance between extreme fiber and resultant of compressive stresses to height of equivalent stress block.
k_1 = ratio of the average compressive stress to $0.85f'_c$.
p = ratio of reinforcement, A_s/bd.
f'_c = 28-day cylinder strength.
b = width of beam.

The ratio k_2/k_1 depends on the assumed distribution of compressive stresses. It equals 1/2 for rectangular distribution, 2/3 for triangular distribution. The computed ultimate moment should not exceed the value given by the following formula:

$$M_u = A_s f_y d \left(1 - \frac{0.59 p f_y}{f'_c}\right) \quad (2a)$$

which can be restated as

$$M_u = f'_c b d^2 q \, (1 - 0.59q) \quad (2b)$$

in which $q = pf_y/f'_c$

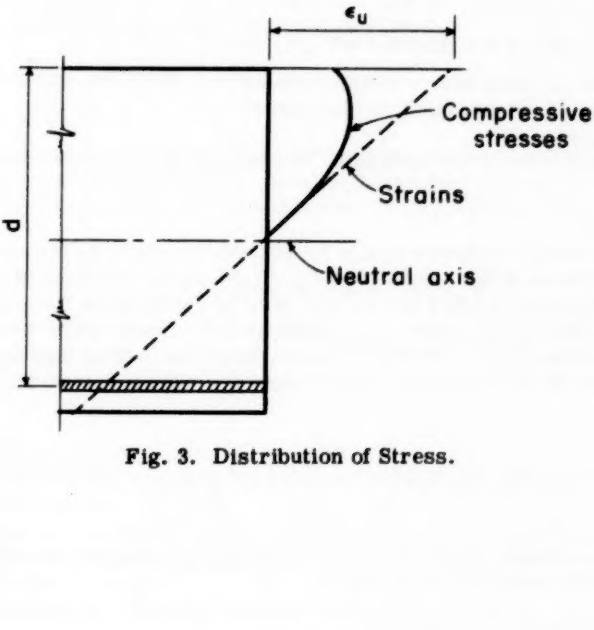

Fig. 3. Distribution of Stress.

In equation (2a) the maximum ratio of reinforcement should be limited to

$$p = \frac{0.40^* f'_c}{f_y} \text{ or } q = 0.40^* \tag{3}$$

This limiting value of p is about 0.9 of that required to develop the full compressive strength of the section.

Rectangular Beams with Compressive Reinforcement:

Stresses in compressive reinforcement at ultimate strength may be assumed equal to the yield point stress or 60,000 psi whichever is the smaller. The resisting ultimate moment should not exceed

$$M_u = (A_s - A'_s)\, f_y d \left[1 - \frac{0.59\,(p-p')f_y}{f'_c}\right] + A'_s f_y^{**}\, (d - d') \tag{4}$$

in which $p-p'$ should not exceed $0.40^* f_c/f_y$ and

A'_s = area of compressive reinforcement
p' = ratio of compressive reinforcement
= A'_s/bd
d' = distance from extreme fiber to centroid of compression steel

T-beams:

When the flange thickness equals or exceeds the depth to the neutral axis given by the formula $k_u d = 1.30\, p f_y d/f'_c = 1.30\, qd$, or the depth of the equivalent stress block (1.18qd) the section may be designed as for a rectangular beam by eq. (2a) or (2b), with p computed as for a beam with a width equal to the overall flange width. When the flange thickness is less than $k_u d$, or less than the depth of the equivalent stress block the resisting moment may be taken as

$$M_u = (A_s - A_{sf}) f_y d \left[1 - \frac{0.59(p_w - p_f)f_y}{f'_c}\right] + A_{sf} f_y\, (d - 0.5\,t) \tag{5}$$

in which A_{sf} = steel area necessary to develop the compressive strength of the overhanging portions of the flange.

A_{sf} = $0.85\, f'_c\, (b-b')\, t/f_y$
t = flange thickness
b = overall width of flange
b' = width of web
p = A_s/bd
p_w = $A_s/b'd$
p_f = $A_{sf}/b'd$

In eq. (5) the value of $(p_w - p_f($ should not exceed $0.40^* f'_c/fy$.

* The coefficient 0.40 is to be reduced at the rate of 0.025 per 1,000 psi concrete strength in excess of 5,000 psi.

** Correction for concrete area displaced by compressive reinforcement may be accounted for by subtracting $0.85\, f'_c$ from f_y.

In T-beam construction, the effective flange width on either side of the web should not be taken greater than six times the thickness of the slab.

Concentrically Loaded Short Columns:

For concentric loads, the maximum load capacity, P_o, is given by the formula

$$P_o = 0.85\, f'_c\, (A_g - A_s) + A_s f_y \tag{6}$$

in which

A_g = the gross area of the section
A_s = cross-sectional area of the bars

However, all members subject to axial loads should be designed for a minimum eccentricity. For spirally reinforced columns the minimum eccentricity measured from the centroidal axis of column should be taken as 0.05 times the depth of the column section. For tied columns the minimum eccentricity should be taken as 0.10 times the depth.

Combined Bending and Axial Load:

Rectangular Section:

The ultimate strength of members subject to combined bending and axial loads should be computed from the usual two equations of equilibrium, which when k_u is less than unity may be expressed as follows:

$$P_u = 0.85 f'_c\, b d k_u k_1 + A'_s f^*_y - A_s f_s \tag{7a}$$

$$P_u e = 0.85 f'_c\, b d^2 k_u k_1 \left(1 - \frac{k_2}{k_1} k_u k_1\right) + A'_s f^*_y d\, (1 - d'/d) \tag{7b}$$

in which

P_u = axial load on the section.
e = eccentricity of the axial load measured from the centroid of tensile reinforcement.
f_s = stress in the tensile reinforcement which equals f_y when tension controls ultimate strength, but is smaller than f_y when compression controls.
$k_u d$ = distance from extreme fiber to neutral axis.

In the above equations k_2/k_1 should not be taken as less than 0.5 and k_1 not greater than 0.85.** When the maximum concrete strain is limited to 0.003 and the modulus of elasticity of the reinforcement is assumed at 30 x 10^6 psi, equation (7a) indicates that the section is controlled by tension when

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*Correction for concrete area displaced by compressive steel area may be accounted for by subtracting $0.85f'_c$ from f_y.

**The coefficient 0.85 and 0.72 is to be reduced at the rate of 0.05 and 0.04 respectively per 1,000 psi concrete strength in excess of 5,000 psi.

$$P_u \lesseqgtr P_b = 0.72^{**}\left(\frac{90{,}000}{90{,}000+f_y}\right) f'_c bd + A'_s f^*_y - A_s f_y \tag{8}$$

When P_u is less than the value given by equation (8) $f_s = f_y$ and taking into account area displaced by the compressive reinforcement, equations (7a) and (7b) reduce to

$$P_u = 0.85 f'_c\, bd \left\{p'm' - pm + 1 - e/d + \sqrt{(1-e/d)^2 + 2\left[\frac{e}{d}\,(pm - p'm') + p'm'\,(1 - d'/d)\right]}\right\} \tag{9}$$

in which

$m \;= f_y/0.85f'_c$
$m' = m-1$
$p \;= A_s/bd$
$p' = A'_s/bd$

For symmetrical reinforcement, equation (9) reduces to

$$P_u = 0.85 f'_c\, bd \left\{-p + 1 - e/d + \sqrt{(1-e/d)^2 + 2p\,\left[m'(1-d'/d) + e/d\right]}\right\} \tag{10}$$

With no compression reinforcement, equation (9) reduces to

$$P_u = 0.85 f'_c\, bd \left[-pm + 1 - e/d + \sqrt{(1-e/d)^2 + 2\,\frac{epm}{d}}\right] \tag{11}$$

When P_u exceeds the value given by equation (8) the ultimate capacity of member is controlled by compression. For this condition, a linear relationship between axial load and moment for values of P_u between that given by equation (8) and a concentric load may be assumed. For this range the ultimate axial load may therefore be computed from the formula:

$$P_u = \frac{P_o}{1 + \left(\frac{P_o}{P_b} - 1\right)\frac{e'}{e'_b}} \tag{12}$$

in which

e' = eccentricity measured from plastic centroid of section.
e'_b = eccentricity of load P_b measured from plastic centroid of section

When the ultimate capacity of a member is controlled by compression, symmetrically reinforced or unsymmetrically reinforced members the equation

$$P_u = \frac{A_s' f_y}{e/(d-d')} + \frac{bt f_c'}{\frac{3t(e-0.5t+d')}{d^2} + 1.18} \tag{13}$$

may be substituted for equations (12).

Circular Section:

The strength of a member of circular cross section subject to combined bending and axial loads may be computed on the basis of the equations of equilibrium taking into account inelastic deformations or by the partially rational and partially empirical formulas (14) and (15).

$$P_u = \frac{A_s f_y}{\frac{3e'}{d} + 1} + \frac{A_g f_c'}{\frac{9.6 De'}{(0.8D + 0.67d)^2} + 1.18} \tag{14}$$

or

$$P_u = 0.85 D^2 f_c' \left[\sqrt{\left(\frac{0.85e'}{D} - 0.377\right)^2 + \frac{p_t m d}{2.5D}} - \left(\frac{0.85e'}{D} - 0.377\right) \right] \tag{15}$$

in which

D = diameter of the column
d = diameter of circle circumscribing the reinforcement
e' = the eccentricity of axial load measured from centroid of section

Slender Members:

When the unsupported length, L, of an axially loaded member is greater than fifteen times its least lateral dimension d, either

1. the effect of the slenderness on the ultimate capacity should be taken into account by stability determination with an apparent reduced modulus of elasticity used for sustained loads. A numerical procedure as that recommended in the Report of Committee 312 "Plain and Reinforced Concrete Arches" Journal of ACI, May 1951 may be used.

or

2. the maximum axial load P'_u should be restricted to

$$P_u' = P_o(1.6 - 0.4L/d) \tag{16}$$

in which P_o is the maximum concentric load capacity of the section with L/d less than fifteen.

Shear and Bond:

Because of the studies and experimental investigations now underway for the joint ACI-ASCE Committee on Shear and Diagonal Tension and for the ACI

Committee on Bond Stress no tentative recommendations regarding the ultimate strength of reinforced concrete members in these two items are proposed at this time.

APPENDIX A

1. Test Data

Evidence of the reliability of design formulas given in the recommendations for the proportioning of sections is presented in Tables 1 to 4 and Fig. 4, inclusive. These tests include most of the available data on the ultimate strength of beams failing in bending and on columns subject to combined bending and axial load. The data covers a wide range of concrete and steel strengths, percentages of reinforcement and specimen sizes.

For the beam specimens the concrete strength ranged from 1390 to approximately 6000 psi, while for the column specimens it varied from 1570 to about 9500 psi. The yield points of the reinforcement ranged from 36,600 and 95,500 psi. The smallest beam was 2 x 3 in. and the largest was a T-beam with a 20-in. wide flange and a web about 12.5 in. deep. Data on the square columns are for 10 x 10-in. and 4 x 4-in. columns. Round columns have a 12-in. diameter.

In Table 1, physical properties are listed in the second to the seventh column, inclusive. The eighth column, headed p_u, is the maximum ratio of reinforcement permitted by equation (3). This does not represent the maximum ratio required to develop the full compressive strength of the concrete.

The ultimate capacity is given in terms of the parameter $M_u/(bd^2f'_c)$. The ninth column is this value computed from the breaking load. The ultimate capacity determined by the conventional straightline procedure is tabulated in the next column. The eleventh column is computed by means of equation (2) except when p exceeds p_u, in which case the ultimate capacity is restricted to the value obtained by p_u. A comparison of test results and computed values expressed as ratios, Test/Straightline or Test/Ultimate, is given in the last two columns.

It should be noted that for ratios of reinforcement below balanced reinforcement, as computed by the straightline procedure, the difference between the straightline and ultimate strength values is small. For larger ratios a significant difference occurs. A graphical representation of values tabulated in Table 1 is given in Fig. 4. The curve has been computed on the basis of equation (2a) expressed in the dimensionless form shown in Fig. 4. The value of p_u is taken from equation (3).

A comparison of test results and computed values for square columns is given in Table 2. The eccentricity of the applied load given in the column headed "e" is the distance from the centroid of the tensile reinforcement.

In contrast to the dimensionless parameter adapted for showing the ultimate capacity of beams, the actual breaking load is tabulated in the twelfth column. Because of the empirical nature of the ACI column formulas, it is not possible to establish the maximum capacity of a column by their procedure. Hence the thirteenth column indicates the design load computed in accordance with provisions of the ACI code. The ratio given in the column headed "Test/ACI" is then the ratio of design load to ultimate and is therefore representative of the safety factor contained in ACI formulas.

P_u is the ultimate capacity as predicted by the ultimate strength theory. Because specimens A1a to C5b are unsymmetrically reinforced, equations (7)

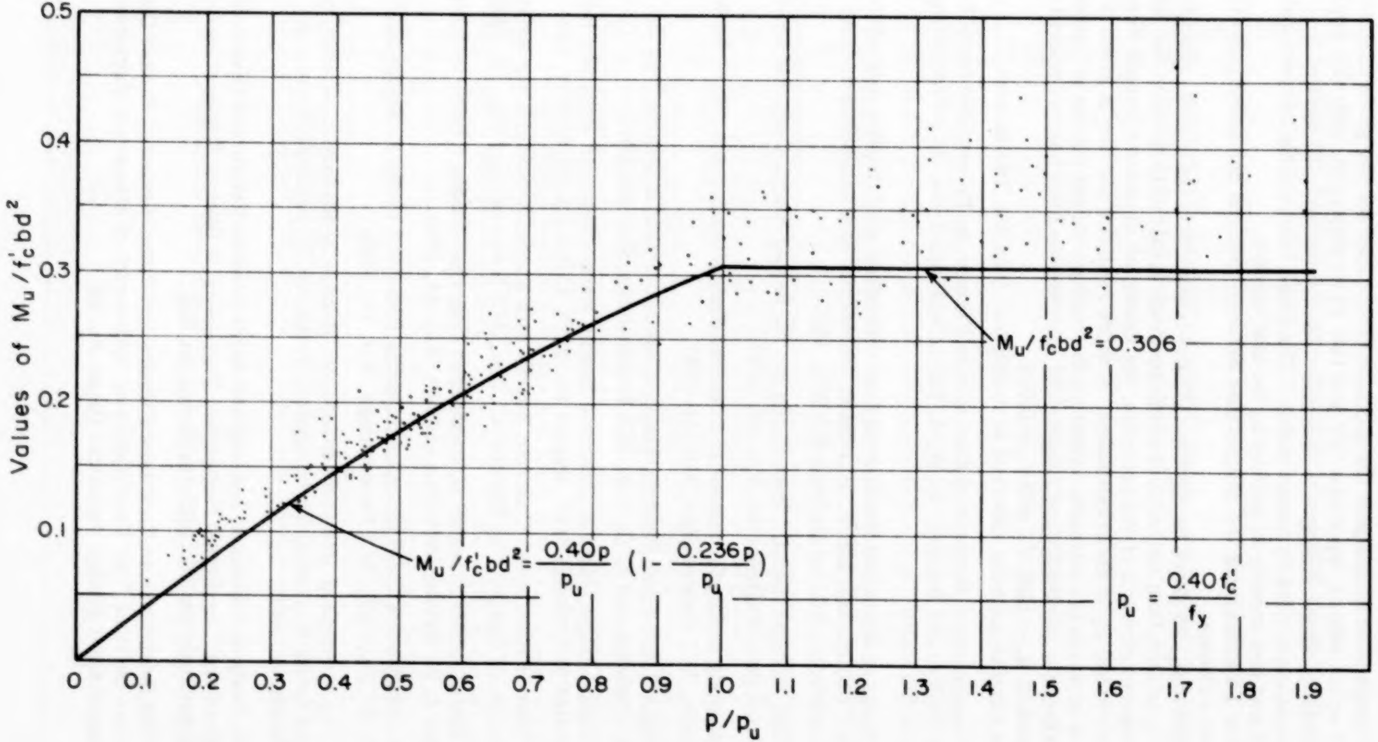

Fig. 4. Computed Ultimate Strength of Concrete Beams Compared with Test Results.

and (9) were used to compute the ultimate direct load for the given eccentricity. For the others, equations (10) and (13) were employed, with (10) applied to specimens whose strength is controlled by tension and (13) applied to specimens controlled by compression. The close relationship between calculated and test results is evident in the last column.

Table 3 summarizes the comparison between test and computed values for circular columns.

Because p_u, as given by equation (3) and tabulated in the eighth column of Table 1, is less than the ratio of reinforcement required to develop the full compressive strength of the concrete, the predicted ultimate strength when p is more than p_u is underestimated. A more realistic picture is given in Table 4 in which the ultimate values by the various stress blocks are given. The values for parabolic and trapezoidal distribution have been computed on the basis of $E_c = 1000 f'_c$ and $\epsilon_u = 0.003$.

The various sources referred to in column '1 of the Tables are:

1. "Compressive Strength of Concrete in Flexure as Determined from Tests of Reinforced Beams," by W. A. Slater and Inge Lyse, ACI Proceedings, Vol. 26, 1930.

2. "Tests of Plain and Reinforced Concrete Made with Haydite Aggregates," by F. E. Richart and V. P. Jensen, University of Illinois Engineering Experiment Station Bulletin No. 237, 1931.

3. "The Modular Ratio," Part III, by Dr. K. Hajnal Konyi, Concrete and Constructional Engineering, Vol. 32, 1937.

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Table No. 1 Comparison of test results with calculated values (beams)

Source	Beam Number	f'_c	f_y	b	d	p	p_u	$M_u/bd^2f'_c$ Test	$M_u/bd^2f'_c$ Straight-line	$M_u/bd^2f'_c$ Ultimate	$\frac{\text{Test}}{\text{Straight-line}}$	$\frac{\text{Test}}{\text{Ultimate}}$
	1	1390	64800	8.2	10.2	0.021	0.0086	0.506	0.235	0.306	2.15	1.65
	2	2790	"	8.2	10.3	0.028	0.0172	0.337	0.141	"	2.38	1.10
	3	4070	"	8.2	10.3	0.037	0.0251	0.326	0.126	"	2.60	1.06
	4	4800	"	8.2	10.1	0.047	0.0296	0.339	0.138	"	2.46	1.11
	5	5740	"	8.3	10.2	0.056	0.0338	0.320	0.137	0.296	2.34	1.08
#1	6	2590	"	8.2	14.2	0.030	0.0160	0.422	0.170	0.306	2.49	1.38
	6A	4130	"	8.2	14.1	0.039	0.0255	0.327	0.131	"	2.50	1.07
	7	2950	"	8.3	12.2	0.028	0.0182	0.341	0.132	"	2.58	1.11
	8	2760	"	8.1	8.0	0.031	0.0170	0.380	0.163	"	2.33	1.24
	9	2900	"	7.9	5.9	0.032	0.0179	0.391	0.159	"	2.45	1.28
	10	2820	"	8.0	4.1	0.030	0.0174	0.346	0.152	"	2.27	1.13
	10A	3810	"	8.0	4.1	0.040	0.0235	0.354	0.150	"	2.36	1.16
	G-1	4730	39100	6	10	0.0103	0.0486	0.092	0.076	0.082	1.21	1.12
	G-2	3870	"	"	"	0.0103	0.0395	0.114	0.093	0.100	1.23	1.14
#2	G-3	3140	"	"	"	0.0103	0.0321	0.138	0.113	0.118	1.22	1.16
	L-1	4830	"	"	"	0.0103	0.0494	0.089	0.075	0.079	1.19	1.12
	L-2	3900	"	"	"	0.0103	0.0399	0.108	0.092	0.097	1.17	1.11
	L-3	2990	"	"	"	0.0103	0.0306	0.144	0.118	0.124	1.22	1.16
	C1	2510	35600	12.3	12.05	0.0062	0.0282	0.106	0.079	0.083	1.34	1.27
	C11	3327	35600	12.0	12.29	0.0062	0.0374	0.078	0.060	0.064	1.30	1.22
	C2	3250	34800	12.2	12.09	0.0062	0.0374	0.074	0.060	0.064	1.22	1.15
	C12	3110	34800	12.0	11.89	0.0064	0.0357	0.081	0.065	0.069	1.25	1.18
#3	C5	3243	35800	12.1	11.96	0.0124	0.0362	0.141	0.120	0.126	1.17	1.11
	C15	3028	35800	12.05	12.16	0.0123	0.0338	0.144	0.127	0.133	1.14	1.08
	D1	3440	36900	19.5	11.04	0.0116	0.0373	0.118	0.110	0.117	1.08	1.01
	D11	2580	36900	19.7	11.14	0.0114	0.0280	0.161	0.141	0.147	1.14	1.09
	C3	3443	58250	12.1	12.13	0.0051	0.0236	0.096	0.080	0.083	1.21	1.17
	C13	3230	58250	12.05	12.00	0.0052	0.0222	0.108	0.085	0.088	1.27	1.22
	C16	3425	54400	12.10	11.87	0.0118	0.0252	0.180	0.159	0.166	1.13	1.08
	D2	3075	55800	19.6	11.0	0.0092	0.0220	0.169	0.148	0.150	1.14	1.12
	D12	3382	55800	19.65	10.8	0.0093	0.0242	0.158	0.137	0.140	1.15	1.13

Table No. 1 Comparison of test results with calculated values (beams)

Source	Beam Number	f'_c*	f_y	b	d	p	p_u	$M_u/bd^2f'_c$ Test	$M_u/bd^2f'_c$ Straight-line	$M_u/bd^2f'_c$ Ultimate	$\frac{\text{Test}}{\text{Straight-line}}$	$\frac{\text{Test}}{\text{Ultimate}}$
#4	2	2645	47000	34	3.12	0.0113	0.0225	0.197	0.171	0.177	1.15	1.11
	3	2565	46000	"	3.06	0.0115	0.0223	0.202	0.174	0.182	1.15	1.11
	5	2555	56600	"	3.08	0.0072	0.0181	0.187	0.139	0.143	1.34	1.30
	6	2645	47500	"	3.00	0.0075	0.0223	0.177	0.120	0.125	1.48	1.42
	7	2510	50000	"	3.12	0.0075	0.0201	0.187	0.133	0.137	1.41	1.37
	9	2745	46000	"	3.10	0.0033	0.0239	0.081	0.051	0.054	1.58	1.50
	10	2655	55000	"	3.40	0.0030	0.0193	0.085	0.058	0.061	1.47	1.40
	11	2470	75000	"	3.06	0.0034	0.0132	0.133	0.093	0.096	1.42	1.39
	12	3220	95500	"	3.00	0.0034	0.0135	0.122	0.094	0.095	1.29	1.28
	21	2945	53100	"	3.00	0.0140	0.0222	0.250	0.177	0.215	1.41	1.16
	22	2985	48500	"	3.10	0.0133	0.0246	0.212	0.173	0.188	1.22	1.12
	23	2925	47100	"	3.06	0.0149	0.0248	0.225	0.181	0.206	1.24	1.09
	24	2955	50300	"	2.94	0.0132	0.0235	0.229	0.173	0.195	1.32	1.17
	25	2930	47000	"	3.06	0.0115	0.0249	0.193	0.161	0.165	1.20	1.17
	26	2850	50000	"	3.00	0.0098	0.0224	0.208	0.151	0.157	1.37	1.33
	28	3040	38950	"	2.75	0.0214	0.0312	0.263	0.190	0.230	1.38	1.14
	29	4995	93000	"	2.94	0.0077	0.0215	0.149	0.131	0.131	1.14	1.14
	31	2735	47900	"	2.94	0.0160	0.0228	0.271	0.188	0.234	1.44	1.16
	32	5460	50000	"	3.12	0.0072	0.0424	0.082	0.061	0.064	1.35	1.28
#5	C1	3200	62200	12	12	0.0022	0.0206	0.062	0.041	0.043	1.51	1.46
	C2	"	59200	"	"	0.0039	0.0216	0.094	0.066	0.069	1.42	1.36
	C3	"	60300	"	"	0.0062	0.0212	0.132	0.105	0.109	1.26	1.22
	C4	"	60300	"	"	0.0082	0.0212	0.172	0.138	0.141	1.25	1.22
	A1 I	"	62200	"	"	0.0045	0.0206	0.103	0.080	0.083	1.29	1.25
	A1 II	"	62200	"	"	0.0045	0.0206	0.104	0.080	0.083	1.30	1.26
	A2 I	"	59200	"	"	0.0039	0.0216	0.097	0.066	0.069	1.47	1.41
	A2 II	"	59200	"	"	0.0039	0.0216	0.096	0.066	0.069	1.46	1.39
	A3 I	"	60300	"	"	0.0041	0.0212	0.096	0.071	0.074	1.35	1.30
	A3 II	"	60300	"	"	0.0041	0.0212	0.095	0.071	0.074	1.34	1.29
	B1 II	"	59200	"	"	0.0078	0.0216	0.151	0.129	0.135	1.18	1.12
	B2 I	"	60300	"	"	0.0082	0.0212	0.148	0.138	0.141	1.08	1.06
	B3 I	"	63200	"	"	0.0083	0.0202	0.175	0.144	0.148	1.22	1.19

* For group #4, f'_c is 0.95 times the strength of 3-in. by 6-in cylinders.

Table No. 1   Comparison of test results with calculated values (beams)

Source	Beam Number	f'_c	f_y	b	d	p	p_u	$M_u/bd^2f'_c$ Test	$M_u/bd^2f'_c$ Straight-line	$M_u/bd^2f'_c$ Ultimate	$\frac{\text{Test}}{\text{Straight-line}}$	$\frac{\text{Test}}{\text{Ultimate}}$
#6	A1	3550	66250	10.03	12.22	0.0049	0.0214	0.111	0.084	0.087	1.33	1.29
	A11	3550	64850	9.92	12.20	0.0050	0.0219	0.112	0.083	0.089	1.34	1.25
	A2	3550	65630	10.03	12.20	0.0048	0.0216	0.110	0.082	0.085	1.35	1.30
	A12	3550	65250	9.98	12.30	0.0049	0.0218	0.110	0.082	0.085	1.34	1.29
	B1	3550	62430	13.98	12.05	0.0144	0.0227	0.255	0.168	0.215	1.52	1.19
	B11	3550	63500	13.95	12.15	0.0143	0.0224	0.269	0.168	0.217	1.60	1.24
	B2	3550	62930	14.05	12.15	0.0141	0.0226	0.236	0.167	0.213	1.41	1.11
	B12	3550	65030	13.98	12.15	0.0142	0.0218	0.244	0.167	0.220	1.46	1.11
	C1	3550	61370	10.48	10.15	0.0341	0.0231	0.406	0.216	0.306	1.88	1.33
	C11	3550	62000	10.43	10.08	0.0345	0.0229	0.394	0.217	0.306	1.82	1.29
	C2	3550	63040	10.45	10.02	0.0334	0.0225	0.386	0.215	0.306	1.80	1.26
	C12	3550	64280	10.60	10.05	0.0328	0.0221	0.365	0.214	0.306	1.71	1.19
	D1	3510	53000	9.95	12.20	0.0050	0.0265	0.097	0.070	0.074	1.40	1.31
	D11	3510	52400	9.98	12.15	0.0051	0.0268	0.102	0.069	0.072	1.48	1.42
	D2	3510	53480	9.98	12.20	0.0049	0.0262	0.099	0.069	0.072	1.45	1.38
	D12	3510	53000	9.90	12.25	0.0049	0.0265	0.097	0.068	0.071	1.42	1.36
	E1	3510	56250	13.95	12.10	0.0138	0.0250	0.212	0.193	0.193	1.10	1.10
	E11	3510	56050	13.90	12.00	0.0141	0.0250	0.211	0.196	0.195	1.08	1.08
	E2	3510	53860	13.95	12.12	0.0144	0.0261	0.210	0.191	0.192	1.10	1.09
	E12	3510	53960	14.00	12.08	0.0144	0.0260	0.209	0.192	0.192	1.09	1.09
#7	121	1700	53400	5.0	5.0	0.0088	0.0127	0.238	0.182	0.231	1.31	1.03
	122	1700	53400	"	"	0.0176	0.0127	0.405	0.220	0.306	1.84	1.32
	123	1700	53400	"	"	0.0264	0.0127	0.423	0.242	0.306	1.75	1.38
	124	1700	53400	"	"	0.0352	0.0127	0.455	0.256	0.306	1.78	1.49
	125	1700	53400	"	"	0.0440	0.0127	0.471	0.266	0.306	1.77	1.54
	141	1700	48100	"	"	0.0120	0.0141	0.288	0.199	0.271	1.45	1.06
	142	1700	48100	"	"	0.0244	0.0141	0.442	0.238	0.306	1.86	1.44
	143	1700	48100	"	"	0.0363	0.0141	0.426	0.258	0.306	1.65	1.39
	144	1700	48100	"	"	0.0488	0.0141	0.490	0.271	0.306	1.81	1.60
	221	3100	53400	"	"	0.0088	0.0232	0.147	0.134	0.138	1.09	1.06

Table No. 1 Comparison of test results with calculated values (beams)

Source	Beam Number	f'_c	f_y	b	d	p	p_u	$M_u/bd^2f'_c$ Test	$M_u/bd^2f'_c$ Straight-line	$M_u/bd^2f'_c$ Ultimate	Test / Straight-line	Test / Ultimate
#7	222-11	3100	53400	5.0	5.0	0.0088	0.0232	0.151	0.134	0.138	1.12	1.09
	222-10	3100	53400	"	"	0.0098	0.0232	0.168	0.149	0.152	1.13	1.10
	222-9	3100	53400	"	"	0.0110	0.0232	0.185	0.166	0.166	1.11	1.11
	222-8	3100	53400	"	"	0.0126	0.0232	0.210	0.168	0.189	1.25	1.11
	222-7	3100	53400	"	"	0.0147	0.0232	0.227	0.177	0.215	1.28	1.05
	222	3100	53400	"	"	0.0176	0.0232	0.249	0.187	0.249	1.33	1.00
	222-5	3100	53400	"	"	0.0220	0.0232	0.308	0.199	0.294	1.55	1.05
	223	3100	53400	"	"	0.0264	0.0232	0.350	0.210	0.306	1.67	1.14
	224	3100	53400	"	"	0.0352	0.0232	0.343	0.225	0.306	1.52	1.12
	225	3100	53400	"	"	0.0440	0.0232	0.353	0.237	0.306	1.49	1.15
	241	3100	48100	"	"	0.0120	0.0258	0.172	0.162	0.166	1.06	1.04
	242	3100	48100	"	"	0.0244	0.0258	0.313	0.205	0.294	1.46	1.07
	243	3100	48100	"	"	0.0368	0.0258	0.370	0.228	0.306	1.62	1.21
	244	3100	48100	"	"	0.0488	0.0258	0.375	0.242	0.306	1.55	1.23
	211	3100	55200	"	"	0.0076	0.0225	0.140	0.121	0.124	1.16	1.13
	212	3100	55200	"	"	0.0156	0.0225	0.257	0.190	0.232	1.35	1.11
	214	3100	55200	"	"	0.0308	0.0225	0.374	0.217	0.306	1.72	1.22
	215	3100	55200	"	"	0.0388	0.0225	0.372	0.230	0.306	1.62	1.22
	231	3100	48100	"	"	0.0076	0.0258	0.127	0.105	0.110	1.21	1.16
	232	3100	48100	"	"	0.0156	0.0258	0.221	0.190	0.207	1.16	1.07
	233	3100	48100	"	"	0.0232	0.0258	0.306	0.200	0.283	1.53	1.08
	234	3100	48100	"	"	0.0308	0.0258	0.348	0.217	0.306	1.60	1.14
	235	3100	48100	"	"	0.0388	0.0258	0.381	0.230	0.306	1.66	1.24
	251	3100	50600	"	"	0.0172	0.0245	0.240	0.187	0.234	1.28	1.02
	252	3100	50600	"	"	0.0348	0.0245	0.379	0.224	0.306	1.69	1.24
	253	3100	50600	"	"	0.0520	0.0245	0.388	0.245	0.306	1.58	1.27
	321	4500	53400	"	"	0.0088	0.0337	0.115	0.094	0.098	1.22	1.17
	322	4500	53400	"	"	0.0176	0.0337	0.190	0.182	0.193	1.04	0.99
	323	4500	53400	"	"	0.0264	0.0337	0.262	0.186	0.255	1.41	1.03
	324	4500	53400	"	"	0.0352	0.0337	0.294	0.205	0.306	1.43	0.96

Table No. 1 Comparison of test results with calculated values (beams)

Source	Beams Number	f'_c	f_y	b	d	p	p_u	$M_u/bd^2f'_c$ Test	$M_u/bd^2f'_c$ Straight-line	$M_u/bd^2f'_c$ Ultimate	$\frac{\text{Test}}{\text{Straight-line}}$	$\frac{\text{Test}}{\text{Ultimate}}$
#7	325	4500	53400	5.0	5.0	0.0440	0.0337	0.320	0.217	0.306	1.47	1.05
	341	4500	48100	"	"	0.0120	0.0374	0.137	0.114	0.119	1.20	1.16
	342	4500	48100	"	"	0.0244	0.0374	0.223	0.188	0.221	1.19	1.01
	343	4500	48100	"	"	0.0368	0.0374	0.297	0.207	0.302	1.43	0.98
	344	4500	48100	"	"	0.0488	0.0374	0.342	0.222	0.306	1.54	1.12
	421	5800	53400	"	"	0.0088	0.0413	0.093	0.074	0.077	1.26	1.20
	422	5800	53400	"	"	0.0176	0.0413	0.146	0.143	0.147	1.02	1.00
	423	5800	53400	"	"	0.0264	0.0413	0.213	0.192	0.208	1.11	1.02
	424	5800	53400	"	"	0.0352	0.0413	0.256	0.190	0.262	1.35	0.98
	425	5800	53400	"	"	0.0440	0.0413	0.293	0.203	0.295	1.44	0.99
	441	5800	48100	"	"	0.0120	0.0458	0.110	0.090	0.094	1.22	1.17
	442	5800	48100	"	"	0.0244	0.0458	0.183	0.176	0.178	1.04	1.03
	443	5800	48100	"	"	0.0368	0.0458	0.256	0.191	0.241	1.34	1.06
	444	5800	48100	"	"	0.0488	0.0458	0.300	0.209	0.295	1.44	1.02
#8	4201	1780	36600	1.99	2.47	0.0118	0.0195	0.246	0.196	0.207	1.26	1.19
	4202	1570	39700	2.00	2.43	0.0156	0.0158	0.349	0.217	0.303	1.61	1.15
	4203	1930	44400	2.00	2.60	0.0230	0.0174	0.390	0.227	0.306	1.72	1.28
	4204	1970	43900	2.00	2.43	0.0305	0.0179	0.463	0.242	0.306	1.91	1.51
	4205	1970	39200	2.04	2.45	0.0400	0.0201	0.449	0.254	0.306	1.77	1.47
	4206	1930	44400	2.03	2.49	0.0475	0.0174	0.464	0.263	0.306	1.76	1.51
	4301	2970	36600	2.03	2.62	0.0110	0.0325	0.143	0.119	0.125	1.21	1.15
	4302	2760	39700	2.02	2.53	0.0150	0.0278	0.202	0.185	0.188	1.09	1.07
	4403	2770	42800	2.00	2.48	0.0230	0.0259	0.317	0.208	0.281	1.53	1.13
	4304	2750	39400	2.01	2.42	0.0238	0.0279	0.308	0.210	0.272	1.47	1.13
	4305	2770	44800	1.98	2.48	0.0305	0.0247	0.382	0.218	0.306	1.75	1.25
	4306	2750	40700	2.10	2.40	0.0298	0.0270	0.351	0.222	0.306	1.58	1.15
	4307	2770	40800	2.04	2.45	0.0398	0.0272	0.440	0.237	0.306	1.86	1.44
	4308	3330	42800	2.02	2.47	0.0454	0.0311	0.367	0.234	0.306	1.57	1.20

Table No. 1 Comparison of test results with calculated values (beams)

Source	Beam Number	f'_c	f_y	b	d	p	p_u	$M_u/bd^2f'_c$ Test	$M_u/bd^2f'_c$ Straight-line	$M_u/bd^2f'_c$ Ultimate	$\frac{\text{Test}}{\text{Straight-line}}$	$\frac{\text{Test}}{\text{Ultimate}}$
	4401	4500	38600	2.00	2.54	0.0114	0.0466	0.107	0.087	0.092	1.22	1.16
	4402	4500	39700	2.02	2.57	0.0145	0.0453	0.121	0.113	0.118	1.07	1.02
	4403	3950	43500	2.00	2.51	0.0231	0.0363	0.229	0.188	0.216	1.22	1.06
	4404	4500	36600	2.04	2.42	0.0235	0.0492	0.167	0.164	0.169	1.02	0.98
	4405	3950	44000	2.01	2.48	0.0300	0.0359	0.294	0.203	0.268	1.45	1.09
	4406	4500	39700	2.00	2.41	0.0316	0.0453	0.240	0.198	0.233	1.21	1.03
	4407	4170	40800	2.07	2.63	0.0367	0.0409	0.305	0.211	0.283	1.44	1.08
	4408	4490	43400	2.02	2.44	0.0471	0.0414	0.346	0.221	0.306	1.56	1.29
	4501	5180	38600	2.00	2.44	0.0119	0.0531	0.095	0.079	0.084	1.19	1.13
	4502	5180	39700	2.02	2.42	0.0157	0.0516	0.113	0.106	0.112	1.06	1.06
	4503	5180	43500	2.00	2.45	0.0232	0.0471	0.184	0.169	0.172	1.09	1.07
	4504	5120	36600	2.00	2.36	0.0246	0.0555	0.163	0.152	0.157	1.07	1.03
	4505	4960	44000	2.00	2.50	0.0296	0.0451	0.229	0.189	0.222	1.21	1.03
	4506	5180	37400	1.99	2.40	0.0318	0.0548	0.209	0.191	0.198	1.09	1.05
	4507	4450	41500	2.05	2.60	0.0374	0.0429	0.297	0.208	0.277	1.43	1.07
	4508	5170	43500	2.04	2.42	0.0466	0.0470	0.308	0.213	0.301	1.45	1.02
#8	6201	1780	58300	2.03	2.48	0.0053	0.0122	0.170	0.152	0.156	1.12	1.09
	6202	2080	86000	2.03	2.51	0.0097	0.0097	0.327	0.176	0.306	1.86	1.07
	6203	2150	88000	2.01	2.69	0.0140	0.0098	0.346	0.194	0.306	1.79	1.13
	6204	2150	75800	2.03	2.69	0.0200	0.0113	0.333	0.214	0.306	1.56	1.09
	6205	1950	75800	2.06	2.46	0.0225	0.0103	0.415	0.226	0.306	1.84	1.36
	6206	2080	73500	2.04	2.58	0.0284	0.0113	0.400	0.234	0.306	1.71	1.31
	6207	1915	75200	2.02	2.50	0.0385	0.0102	0.391	0.254	0.306	1.54	1.28
	6208	2120	75200	2.02	2.45	0.0391	0.0113	0.407	0.250	0.306	1.63	1.33
	6301	2750	62500	2.01	2.42	0.0055	0.0176	0.115	0.115	0.113	1.02	0.99
	6302	3200	83500	2.00	2.65	0.0093	0.0153	0.217	0.151	0.208	1.44	1.04
	6303	3290	72000	2.00	2.58	0.0147	0.0183	0.270	0.173	0.260	1.56	1.04
	6304	2760	75800	2.00	2.45	0.0233	0.0146	0.347	0.208	0.306	1.67	1.13
	6305	3200	74000	2.01	2.60	0.0286	0.0173	0.338	0.212	0.306	1.59	1.10
	6306	2760	75200	2.00	2.47	0.0394	0.0147	0.359	0.237	0.306	1.51	1.17

Table No. 1 Comparison of test results with calculated values (beams)

Source	Beam Number	f'_c*	f_y	b	d	p	p_u	$M_u/bd^2f'_c$ Test	$M_u/bd^2f'_c$ Straight-line	$M_u/bd^2f'_c$ Ultimate	$\frac{\text{Test}}{\text{Straight-line}}$	$\frac{\text{Test}}{\text{Ultimate}}$
	6401	4490	64000	2.00	2.43	0.0055	0.0281	0.077	0.071	0.075	1.08	1.04
	6402	4140	82000	2.04	2.67	0.0090	0.0202	0.151	0.135	0.159	1.12	0.95
	6403	4140	81500	2.00	2.61	0.0151	0.0203	0.268	0.165	0.245	1.62	1.09
	6404	4490	75800	2.10	2.40	0.0226	0.0237	0.263	0.190	0.295	1.38	0.89
	6405	4140	74000	2.06	2.58	0.0280	0.0224	0.293	0.196	0.306	1.49	0.96
	6406	4190	75200	2.01	2.47	0.0390	0.0223	0.317	0.214	0.306	1.48	1.04
#8	6407	4190	62100	2.00	2.47	0.0408	0.0270	0.314	0.215	0.306	1.46	1.03
	6501	4450	86000	1.99	2.68	0.0092	0.0207	0.166	0.136	0.159	1.22	1.05
	6502	4450	88000	2.03	2.58	0.0144	0.0202	0.265	0.156	0.237	1.69	1.12
	6503	4770	65000	1.98	2.45	0.0230	0.0294	0.286	0.178	0.255	1.61	1.22
	6504	4870	75800	2.01	2.43	0.0233	0.0257	0.247	0.192	0.285	1.29	0.87
	6505	4450	65000	1.99	2.64	0.0371	0.0274	0.299	0.207	0.306	1.44	0.98
	6506	5450	75800	2.01	2.48	0.0458	0.0279	0.292	0.208	0.299	1.40	0.97
	1110	2240	38200	8.1	9.56	0.0304	0.0235	0.347	0.234	0.306	1.48	1.14
	1112	2210	"	"	"	"	0.0231	0.414	0.235	"	1.76	1.35
	1113	2040	"	"	"	"	0.0214	0.316	0.239	"	1.32	1.03
	1116	2740	"	"	"	"	0.0287	0.364	0.223	"	1.63	1.19
	1119	2510	"	"	"	"	0.0263	0.281	0.228	"	1.23	0.92
	1120	2400	"	"	"	"	0.0251	0.280	0.231	"	1.21	0.92
	1121	2100	"	"	"	"	0.0220	0.283	0.238	"	1.19	0.92
	1123	2260	"	"	"	"	0.0237	0.351	0.234	"	1.50	1.15
	1125	2020	"	"	"	"	0.0212	0.367	0.240	"	1.53	1.20
#9	1126	2070	"	"	"	"	0.0217	0.327	0.239	"	1.37	1.07
	1128	2350	"	"	"	"	0.0246	0.368	0.232	"	1.59	1.20
	1129	2530	"	"	"	"	0.0265	0.344	0.228	"	1.51	1.12
	1130	2450	"	"	"	"	0.0257	0.341	0.229	"	1.49	1.11
	1131	2740	"	"	"	"	0.0287	0.265	0.223	"	1.19	0.87
	1132	2700	"	"	"	"	0.0282	0.285	0.224	"	1.27	0.93
	1133	2720	"	"	"	"	0.0285	0.286	0.224	"	1.28	0.93
	1134	2840	"	"	"	"	0.0297	0.325	0.221	"	1.47	1.06
	1135	2790	"	"	"	"	0.0292	0.310	0.223	"	1.39	1.01

* For group #9, f'_c is 1.05 times the strength of 8-in. by 16-in. cylinders.

Table No. 1 Comparison of test results with calculated values (beams)

Source	Beam Number	f'_c	f_y	b	d	p	p_u	$M_u/bd^2f'_c$ Test	$M_u/bd^2f'_c$ Straight-line	$M_u/bd^2f'_c$ Ultimate	Test / Straight-line	Test / Ultimate
	1136	2850	38200	8.1	9.56	0.0304	0.0298	0.284	0.221	0.306	1.28	0.93
	1137	2630	"	"	"	"	0.0275	0.296	0.226	"	1.31	0.97
	1138	2500	"	"	"	"	0.0262	0.318	0.228	"	1.39	1.04
#9	1139	2210	"	"	"	"	0.0231	0.352	0.235	"	1.50	1.15
	1140	3050	"	"	"	"	0.0319	0.340	0.218	0.295	1.56	1.15
	1141	2890	"	"	"	"	0.0303	0.348	0.220	0.306	1.58	1.14
	1142	2960	"	"	"	"	0.0310	0.361	0.219	0.301	1.65	1.20
	210	3030	42220	8.0	9.25	0.0159	0.0287	0.187	0.183	0.193	1.02	0.97
	211	2610	"	"	"	"	0.0247	0.199	0.191	0.218	1.04	0.91
	212	3100	"	"	"	"	0.0293	0.168	0.181	0.189	0.92	0.89
	222	3570	42100	"	9.36	0.0183	0.0339	0.176	0.181	0.188	0.97	0.94
	223	3490	42020	"	"	"	0.0332	0.185	0.183	0.192	1.01	0.96
	224	3200	42300	"	"	"	0.0303	0.191	0.187	0.207	1.02	0.92
	237	3510	41650	"	9.25	0.0212	0.0337	0.198	0.190	0.214	1.04	0.92
	238	3200	41720	"	"	"	0.0307	0.207	0.195	0.231	1.06	0.90
	239	3260	42200	"	"	"	0.0309	0.214	0.194	0.231	1.10	0.93
	234	5200	41460	"	"	"	0.0495	0.141	0.147	0.152	0.96	0.93
	235	5320	42440	"	"	"	0.0492	0.144	0.147	0.152	0.98	0.95
	236	5130	41730	"	"	"	0.0488	0.143	0.150	0.154	0.95	0.93
#10	297	2990	40330	"	"	0.0159	0.0296	0.174	0.182	0.187	0.96	0.93
	298	3150	40790	"	"	"	0.0309	0.166	0.177	0.180	0.94	0.92
	299	2290	40660	"	"	"	0.0225	0.212	0.198	0.235	1.07	0.90
	318	2790	42620	"	9.36	0.0183	0.0262	0.207	0.194	0.233	1.07	0.89
	319	2670	39740	"	"	"	0.0269	0.204	0.197	0.228	1.04	0.89
	320	1950	40900	"	"	"	0.0190	0.279	0.216	0.297	1.29	0.94
	321	3010	39210	"	9.25	0.0212	0.0307	0.208	0.199	0.231	1.04	0.90
	322	3050	37140	"	"	"	0.0328	0.205	0.198	0.219	1.04	0.94
	323	2550	37760	"	"	"	0.0270	0.260	0.208	0.256	1.25	1.03
	312	4180	41120	"	9.36	0.0183	0.0407	0.147	0.156	0.161	0.94	0.91
	313	3750	40070	"	"	"	0.0374	0.173	0.169	0.173	1.02	1.00
	314	3540	40040	"	"	"	0.0354	0.180	0.177	0.181	1.02	0.91

Table No. 1 Comparison of test results with calculated values (beams)

Source	Beam Number	f'_c	f_y	b	d	p	Pu	$M_u/bd^2f'_c$ Test	$M_u/bd^2f'_c$ Straight-line	$M_u/bd^2f'_c$ Ultimate	Test / Straight-line	Test / Ultimate
	324	4030	37250	8.0	9.25	0.0212	0.0433	0.190	0.168	0.173	1.13	1.10
	325	4010	37730	"	"	"	0.0425	0.183	0.171	0.176	1.07	1.04
	326	3900	36830	"	"	"	0.0424	0.170	0.171	0.176	0.99	0.97
	315	3980	39600	"	9.36	0.0183	0.0402	0.163	0.157	0.163	1.04	1.00
	316	4120	37530	"	"	"	0.0439	0.157	0.144	0.150	1.09	1.05
	317	4000	39870	"	"	"	0.0401	0.162	0.158	0.162	1.02	1.00
	327	4090	36730	"	9.25	0.0212	0.0445	0.162	0.163	0.171	0.99	0.95
	328	4080	37140	"	"	0.0212	0.0439	0.171	0.166	0.171	1.03	1.00
	330	4870	36790	"	9.25	0.0212	0.0529	0.150	0.139	0.145	1.08	1.03
	331	4010	37510	"	"	"	0.0428	0.191	0.170	0.174	1.12	1.10
#10	332	4480	38720	"	"	"	0.0463	0.171	0.158	0.163	1.08	1.05
	381	2790	38010	"	9.25	0.0159	0.0294	0.186	0.185	0.188	1.01	0.99
	382	3240	38570	"	"	"	0.0336	0.171	0.162	0.168	1.06	1.02
	383	3490	39170	"	"	"	0.0356	0.159	0.154	0.160	1.03	0.99
	393	3400	39680	"	9.36	0.0183	0.0343	0.180	0.183	0.187	0.98	0.96
	394	3010	39120	"	"	"	0.0308	0.203	0.190	0.204	1.07	1.00
	395	2920	40030	"	"	"	0.0292	0.209	0.194	0.214	1.08	1.02
	405	3750	38630	"	9.25	0.0212	0.0388	0.176	0.186	0.190	0.95	0.93
	406	3380	38680	"	"	"	0.0350	0.206	0.194	0.208	1.06	0.99
	407	3530	38590	"	"	"	0.0365	0.197	0.190	0.200	1.04	0.99
	397	4140	41020	"	9.36	0.0183	0.0404	0.148	0.157	0.162	0.94	0.91
	398	3990	38920	"	"	"	0.0410	0.162	0.155	0.160	1.04	1.01
	409	3790	38630	"	9.25	0.0212	0.0393	0.202	0.185	0.188	1.09	1.07
	410	4190	38110	"	"	"	0.0440	0.174	0.166	0.171	1.05	1.02
	414	5570	38670	"	9.25	0.0212	0.0555	0.131	0.129	0.134	1.02	0.98
	415	5840	38320	"	"	"	0.0578	0.125	0.122	0.128	1.02	0.98
	416	5850	38940	"	"	"	0.0569	0.125	0.123	0.129	1.02	0.97

Table No. 1 Comparison of test results with calculated values (beams)

Source	Beam Number	f'_c	f_y	b	d	p	Pu	$M_u/bd^2f'_c$			Test / Straight-line	Test / Ultimate
								Test	Straight-line	Ultimate		
	468	2320	38010	8.0	9.25	0.0159	0.0244	0.222	0.197	0.221	1.13	1.00
	469	2295	37970	"	"	"	0.0242	0.225	0.197	0.222	1.14	1.01
	470	2320	37630	"	"	"	0.0246	0.222	0.197	0.219	1.13	1.01
	480	2260	36110	"	9.36	0.0183	0.0250	0.238	0.206	0.242	1.16	0.98
	481	2220	36260	"	"	"	0.0245	0.244	0.207	0.246	1.18	0.99
	482	2230	36980	"	"	"	0.0241	0.256	0.207	0.249	1.24	1.03
#10	492	2220	37210	"	9.25	0.0212	0.0239	0.264	0.216	0.280	1.22	0.94
	494	2280	36440	"	"	"	0.0250	0.257	0.214	0.268	1.20	0.96
	471	2850	38100	"	9.25	0.0159	0.0299	0.181	0.182	0.186	0.99	0.97
	472	2750	36800	"	"	"	0.0299	0.187	0.181	0.186	1.03	1.01
	473	2730	37940	"	"	"	0.0288	0.188	0.188	0.192	1.00	0.98
	483	2400	35920	"	9.36	0.0183	0.0267	0.238	0.203	0.230	1.17	1.03
	484	2470	36460	"	"	"	0.0271	0.246	0.201	0.227	1.22	1.08
	485	2300	36880	"	"	"	0.0249	0.234	0.206	0.243	1.14	0.96
	495	2700	37140	"	9.25	0.0212	0.0291	0.244	0.205	0.241	1.19	1.01
	496	2510	36790	"	"	"	0.0273	0.261	0.209	0.253	1.25	1.03
	497	2610	37820	"	"	"	0.0276	0.265	0.207	0.252	1.28	1.01
	474	2830	36710	"	9.25	0.0159	0.0308	0.170	0.176	0.181	0.97	0.95
	475	2930	38770	"	"	"	0.0302	0.188	0.180	0.184	1.04	1.02
	476	2920	37400	"	"	"	0.0312	0.164	0.176	0.179	0.93	0.92
	486	2910	37620	"	9.36	0.0183	0.0309	0.185	0.192	0.204	0.96	0.91
	487	2740	37230	"	"	"	0.0294	0.221	0.195	0.212	1.13	1.04
	488	2800	37230	"	"	"	0.0301	0.204	0.194	0.208	1.05	1.03
	498	3020	37760	"	9.25	0.0212	0.0320	0.217	0.198	0.223	1.10	0.97
	499	2920	38620	"	"	"	0.0303	0.225	0.199	0.234	1.13	0.96
	500	2780	37760	"	"	"	0.0294	0.236	0.202	0.239	1.17	0.99

Table No. 2 Comparison of test results with calculated values (square columns)

Source	Column Number	f'_c	f_y	b	d	p	p'	e	e/d	e'/t	P_u Test	P ACI	P_u Ultimate	$\frac{\text{Test}}{\text{ACI}}$	$\frac{\text{Test}}{\text{Ultimate}}$
	A-1a	5280	43600*	10.0	8.67	0.0143	0.0025	3.79	0.437	0.012	388.0	105.8	474.0	3.67	0.82
	b	5660	60000**	"	"	"	"	3.81	0.439	0.014	441.0	103.0	474.0	4.28	0.93
	B-1a	4250	"	"	"	"	"	3.79	0.437	0.012	343.0	87.6	386.0	3.91	0.89
	b	4070	"	"	"	"	"	3.79	0.437	0.012	352.0	84.0	370.0	4.19	0.95
	C-1a	2270	"	"	"	"	"	3.80	0.438	0.013	222.0	51.9	224.0	4.28	0.99
	b	2020	"	"	"	"	"	3.80	0.438	0.013	191.0	47.0	203.0	4.07	0.94
	A-2a	5280	"	"	"	"	"	6.39	0.737	0.272	239.0	63.0	251.0	3.80	0.95
	b	5830	"	"	"	"	"	6.45	0.744	0.278	253.0	69.0	271.0	3.66	0.93
	B-2a	4250	"	"	"	"	"	6.44	0.743	0.277	213.0	52.2	205.0	4.08	1.04
	b	4070	"	"	"	"	"	6.41	0.739	0.274	190.0	50.7	198.0	3.75	0.96
	C-2a	2270	"	"	"	"	"	6.44	0.743	0.277	118.5	30.5	119.0	3.89	1.00
	b	1970	"	"	"	"	"	6.44	0.743	0.277	100.0	26.9	106.0	3.72	0.94
	A-3a	5660	"	"	"	"	"	8.99	1.037	0.532	133.5	48.9	155.6	2.73	0.86
	b	5830	"	"	"	"	"	8.95	1.032	0.528	140.0	50.4	160.0	2.78	0.88
#11	B-3a	4630	"	"	"	"	"	9.08	1.047	0.541	125.9	41.1	135.0	3.06	0.93
	b	4290	"	"	"	"	"	9.04	1.043	0.537	116.0	37.8	131.0	3.07	0.89
	C-3a	1880	"	"	"	"	"	8.95	1.032	0.528	60.5	18.8	74.0	3.22	0.82
	b	1690	"	"	"	"	"	9.00	1.038	0.533	64.0	17.1	65.0	3.74	0.98
	A-4a	4810	"	"	"	"	"	11.62	1.340	0.795	84.5	33.2	86.0	2.55	0.98
	b	5600	"	"	"	"	"	11.52	1.329	0.785	81.0	37.9	93.0	2.14	0.87
	B-4a	3800	"	"	"	"	"	11.65	1.344	0.798	80.0	26.3	78.0	3.04	1.03
	b	4290	"	"	"	"	"	11.69	1.348	0.802	81.0	29.5	79.0	2.75	1.02
	C-4a	1690	"	"	"	"	"	11.49	1.325	0.782	50.5	13.4	47.0	3.77	1.08
	b	1730	"	"	"	"	"	11.48	1.325	0.781	52.0	13.7	46.0	3.80	1.13
	A-5a	4810	"	"	"	"	"	16.57	1.911	1.290	48.2	16.3	45.0	2.96	1.06
	b	5600	"	"	"	"	"	16.57	1.911	1.290	42.8	16.6	46.0	2.58	0.93
	B-5a	4290	"	"	"	"	"	16.59	1.913	1.292	46.1	15.3	44.0	3.01	1.04
	b	4590	"	"	"	"	"	16.62	1.917	1.295	45.5	16.0	44.0	2.84	1.03
	C-5a	2310	"	"	"	"	"	16.51	1.904	1.284	39.0	10.0	37.0	3.90	1.05
	b	1770	"	"	"	"	"	16.51	1.904	1.284	32.8	8.9	33.0	3.69	0.98

* Tension reinforcement
** Compression reinforcement

Table No. 2 Comparison of test results with calculated values (square columns)

Source	Column Number	f'_c	f_y	b	d	p	p'	e	e/d	e'/t	P_u Test	P ACI	P_u Ultimate	Test / ACI	Test / Ultimate
	B-6a	4080	43600	10.0	8.67	0.0143	0.0143	3.74	0.431	0.007	456.0	103.2	443.0	4.42	1.03
	b	4040	"	"	"	"	"	3.73	0.430	0.006	420.0	102.8	442.0	4.09	0.95
	C-6a	2020	"	"	"	"	"	3.77	0.434	0.010	225.0	66.1	272.0	3.40	0.83
	b	1520	"	"	"	"	"	3.85	0.444	0.018	202.0	56.0	224.0	3.61	0.90
	A-7a	5240	"	"	"	"	"	7.11	0.820	0.344	274.0	65.0	261.0	4.22	1.05
	b	5810	"	"	"	"	"	6.43	0.742	0.276	284.0	78.3	316.0	3.63	0.90
	B-7a	4080	"	"	"	"	"	6.42	0.740	0.275	256.0	59.7	241.0	4.29	1.06
	b	4040	"	"	"	"	"	6.41	0.739	0.274	248.0	59.3	240.0	4.18	1.04
	C-7a	1970	"	"	"	"	"	6.45	0.744	0.278	141.0	36.6	148.0	3.85	0.96
	b	1520	"	"	"	"	"	6.46	0.745	0.277	127.0	31.9	128.0	3.97	0.99
	A-8a	5520	"	"	"	"	"	9.01	1.039	0.534	162.0	53.8	175.0	3.01	0.93
	b	5810	"	"	"	"	"	9.07	1.046	0.540	152.0	55.5	174.0	2.74	0.87
	B-8a	4700	"	"	"	"	"	9.02	1.040	0.535	156.0	47.5	162.0	3.28	0.96
	b	4260	"	"	"	"	"	8.99	1.034	0.532	146.0	43.9	156.0	3.33	0.94
#11	C-8a	1820	"	"	"	"	"	8.99	1.037	0.532	99.0	24.6	99.3	4.02	1.00
	b	1820	"	"	"	"	"	9.06	1.045	0.539	99.0	24.6	98.1	4.02	1.01
	A-9a	5100	"	"	"	"	"	11.54	1.331	0.787	89.0	39.4	98.1	2.26	0.91
	b	5170	"	"	"	"	"	11.56	1.333	0.789	91.2	39.6	97.9	2.30	0.93
	B-9a	4700	"	"	"	"	"	11.52	1.329	0.785	94.0	36.9	96.6	2.55	0.97
	b	4370	"	"	"	"	"	11.49	1.325	0.782	89.5	34.8	95.7	2.57	0.94
	C-9a	1880	"	"	"	"	"	11.55	1.332	0.788	73.0	19.5	74.8	3.74	0.98
	b	1730	"	"	"	"	"	11.52	1.329	0.785	65.5	18.5	73.2	3.54	0.89
	A-10a	5100	"	"	"	"	"	16.45	1.897	1.278	46.1	16.2	48.1	2.84	0.96
	b	5170	"	"	"	"	"	16.42	1.894	1.275	44.0	16.3	48.0	2.70	0.92
	B-10a	4260	"	"	"	"	"	16.45	1.897	1.278	43.5	16.6	47.7	2.62	0.91
	b	4370	"	"	"	"	"	16.46	1.897	1.279	44.0	16.6	46.7	2.65	0.94
	C-10a	2300	"	"	"	"	"	16.52	1.905	1.285	44.5	16.2	44.1	2.75	1.01
	b	1770	"	"	"	"	"	16.55	1.909	1.288	45.0	14.5	42.0	3.10	1.07

Table No. 2 Comparison of test results with calculated values (square columns)

Source	Column Number	f'_c	f_y	b	d	p	p'	e	e/d	e'/t	P_u Test	P ACI	P_u Ultimate	$\frac{\text{Test}}{\text{ACI}}$	$\frac{\text{Test}}{\text{Ultimate}}$
	B-11a	3870	43600	10.0	8.5	0.0282	0.0282	3.58	0.421	0.008	500.0	127.9	524.0	3.91	0.95
	b	4010	"	"	"	"	"	3.60	0.423	0.010	485.0	129.6	532.0	3.74	0.91
	C-11b	2070	"	"	"	"	"	3.50	0.412	0.000	353.0	98.7	385.0	3.58	0.92
	A-12a	4150	"	"	"	"	"	6.20	0.729	0.270	315.0	74.4	298.0	4.23	1.06
	b	5050	"	"	"	"	"	6.22	0.732	0.272	325.0	83.6	336.0	3.89	0.97
	B-12a	4300	"	"	"	"	"	6.22	0.731	0.272	303.0	75.6	304.0	4.01	1.00
	b	4010	"	"	"	"	"	6.26	0.737	0.276	284.0	72.2	289.0	3.93	0.98
	C-12a	2300	"	"	"	"	"	6.26	0.736	0.276	252.0	54.2	216.0	4.65	1.17
	b	2200	"	"	"	"	"	6.22	0.731	0.272	230.0	53.8	213.0	4.28	1.08
	A-13a	5350	"	"	"	"	"	8.86	1.042	0.536	220.0	60.3	240.0	3.65	0.92
	b	4850	"	"	"	"	"	8.84	1.040	0.534	210.0	56.7	226.0	3.70	0.93
	B-13a	3580	"	"	"	"	"	8.85	1.041	0.535	180.0	47.4	188.0	3.80	0.96
	b	4290	"	"	"	"	"	8.84	1.040	0.534	206.0	52.7	209.0	3.91	0.98
#11	C-13a	2300	"	"	"	"	"	8.83	1.039	0.533	151.0	37.6	151.0	4.02	1.00
	b	2070	"	"	"	"	"	8.28	0.974	0.528	137.0	35.8	154.0	3.83	0.89
	A-14a	5350	"	"	"	"	"	11.37	1.338	0.787	142.0	46.6	155.0	3.05	0.92
	b	5100	"	"	"	"	"	11.43	1.345	0.793	153.0	45.4	152.0	3.37	1.01
	B-14a	3580	"	"	"	"	"	11.39	1.340	0.789	138.8	37.2	137.0	3.73	1.01
	C-14a	1950	"	"	"	"	"	11.34	1.334	0.784	115.5	26.5	109.0	4.36	1.06
	b	2070	"	"	"	"	"	11.37	1.338	0.787	104.0	27.3	111.0	3.81	0.94
	A-15a	5100	"	"	"	"	"	16.42	1.932	1.292	88.0	29.6	82.2	2.97	1.07
	b	4850	"	"	"	"	"	16.35	1.924	1.285	79.0	29.6	82.7	2.67	0.96
	B-15a	3800	"	"	"	"	"	16.41	1.931	1.291	74.0	27.2	79.4	2.72	0.93
	b	4630	"	"	"	"	"	16.42	1.932	1.292	84.5	28.9	83.0	2.92	1.02
	C-15a	1950	"	"	"	"	"	16.39	1.928	1.289	72.5	23.0	74.6	3.15	0.97
	b	2070	"	"	"	"	"	16.41	1.931	1.291	74.5	23.4	72.3	3.18	1.03

Table No. 2 Comparison of test results with calculated values (square columns)

Source	Column Number	f'_c	f_y	b	d	p	p'	e	e/d	e'/t	P_u Test	P ACI	P_u Ultimate	$\frac{\text{Test}}{\text{ACI}}$	$\frac{\text{Test}}{\text{Ultimate}}$
	8	8325	46600	4.0	3.25	0.0077	0.0077	2.65	0.815	0.350	51.45	14.00	51.74	3.67	0.99
	9	8210	"	"	"	"	"	3.65	1.123	0.600	12.00	10.29	16.69	1.17	0.72
	10	8210	"	"	"	"	"	4.65	1.431	0.850	7.00	8.20	7.70	0.85	0.91
	11	8210	"	"	"	"	"	5.65	1.738	1.100	4.25	1.68	4.80	2.51	0.89
	12	8210	"	"	"	"	"	6.65	2.046	1.350	3.53	1.25	3.81	2.82	0.93
	13	8210	"	"	"	"	"	7.65	2.354	1.600	2.60	1.00	2.90	2.60	0.89
	14	8210	"	"	"	"	"	8.65	2.662	1.850	2.20	0.82	2.18	2.68	1.01
	22	6540	"	"	"	"	"	2.65	0.815	0.350	42.00	11.22	42.10	3.74	1.00
	23	6540	"	"	"	"	"	3.65	1.123	0.600	12.10	8.37	15.54	1.45	0.78
	24	6540	"	"	"	"	"	4.65	1.431	0.850	6.61	6.69	7.59	0.99	0.87
	25	6375	"	"	"	"	"	5.65	1.738	1.100	4.52	1.66	4.79	2.72	0.94
	26	6375	"	"	"	"	"	6.65	2.046	1.350	3.21	1.23	3.38	2.61	0.95
	27	5540	"	"	"	"	"	7.65	2.354	1.600	2.58	0.95	2.63	2.72	0.98
#12	28	6065	"	"	"	"	"	8.65	2.662	1.850	2.14	0.80	2.14	2.68	1.00
	50	7985	"	"	"	0.012	0.012	2.65	0.815	0.350	55.70	13.95	52.88	4.00	1.05
	51	7990	"	"	"	"	"	3.65	1.123	0.600	20.70	10.38	22.34	2.00	0.93
	52	7990	"	"	"	"	"	4.65	1.431	0.850	11.30	8.28	11.48	1.36	0.98
	53	7220	"	"	"	"	"	5.65	1.738	1.100	8.00	2.49	7.34	3.21	1.09
	54	7600	"	"	"	"	"	6.65	2.046	1.350	4.37	1.85	5.37	2.36	0.81
	55	7600	"	"	"	"	"	7.65	2.354	1.600	3.80	1.48	4.20	2.56	0.90
	56	8150	"	"	"	"	"	8.65	2.662	1.850	3.36	1.24	3.42	2.71	0.98
	64	6775	"	"	"	"	"	2.65	0.815	0.350	50.60	12.08	45.87	4.20	1.10
	65	6695	"	"	"	"	"	3.65	1.123	0.600	21.30	8.91	20.93	2.39	1.02
	66	6695	"	"	"	"	"	4.65	1.431	0.850	11.00	7.04	11.14	1.56	0.99
	67	6695	"	"	"	"	"	5.65	1.738	1.100	7.53	2.48	7.23	3.04	1.04
	68	6695	"	"	"	"	"	6.65	2.046	1.350	5.47	1.87	5.02	2.92	1.09
	69	6695	"	"	"	"	"	7.65	2.354	1.600	4.52	1.43	4.28	3.16	1.06
	70	6695	"	"	"	"	"	8.65	2.662	1.850	3.80	1.21	3.40	3.13	1.12

Table No. 2 Comparison of test results with calculated values (square columns)

Source	Column Number	f'_c	f_y	b	d	p	p'	e	e/d	e'/t	P_u Test	P ACI	P_u Ultimate	Test / ACI	Test / Ultimate
	92	7600	46600	4.0	3.25	0.0169	0.0169	2.65	0.815	0.350	54.10	13.92	53.62	3.88	1.01
	93	8020	"	"	"	"	"	3.65	1.123	0.600	26.25	10.88	27.74	2.42	0.95
	94	8020	"	"	"	"	"	4.65	1.431	0.850	14.60	8.58	15.33	1.70	0.95
	95	8020	"	"	"	"	"	5.65	1.738	1.100	8.60	3.50	10.10	2.46	0.85
	96	8020	"	"	"	"	"	6.65	2.046	1.350	6.50	2.46	7.26	2.64	0.90
	97	8020	"	"	"	"	"	7.65	2.354	1.600	5.39	2.03	5.67	2.65	0.95
	98	8960	"	"	"	"	"	8.65	2.662	1.850	4.90	1.74	4.85	2.81	1.01
	106	6065	"	"	"	"	"	2.65	0.815	0.350	47.50	11.62	44.75	4.08	1.06
	107	6065	"	"	"	"	"	3.65	1.123	0.600	23.40	8.57	24.93	2.74	0.94
	108	6065	"	"	"	"	"	4.65	1.431	0.850	14.50	6.77	14.61	2.14	0.99
	109	6065	"	"	"	"	"	5.65	1.738	1.100	10.40	3.29	9.78	3.16	1.06
	110	6065	"	"	"	"	"	6.65	2.046	1.350	8.35	2.56	7.24	3.26	1.15
	111	6065	"	"	"	"	"	7.65	2.354	1.600	5.15	2.04	5.76	2.52	0.89
#12	112	5790	"	"	"	"	"	8.65	2.662	1.850	4.50	1.67	4.74	2.70	0.95
	134	7560	"	"	"	0.0231	0.0231	2.65	0.815	0.350	61.90	14.69	56.89	4.21	1.09
	135	7560	"	"	"	"	"	3.65	1.123	0.600	29.45	10.80	32.75	2.73	0.89
	136	7560	"	"	"	"	"	4.65	1.431	0.850	18.40	8.55	19.63	2.15	0.94
	137	7560	"	"	"	"	"	5.65	1.738	1.100	12.10	4.52	13.37	2.68	0.91
	138	7560	"	"	"	"	"	6.65	2.046	1.350	9.30	3.40	9.94	2.74	0.94
	139	7560	"	"	"	"	"	7.65	2.354	1.600	7.32	2.72	7.85	2.69	0.93
	140	7560	"	"	"	"	"	8.65	2.662	1.850	6.07	2.27	6.52	2.67	0.93
	148	4810	"	"	"	"	"	2.65	0.815	0.350	42.35	10.41	41.00	4.07	1.03
	149	4810	"	"	"	"	"	3.65	1.123	0.600	24.00	7.62	22.75	3.15	1.05
	150	4810	"	"	"	"	"	4.65	1.431	0.850	16.10	6.00	14.99	2.68	1.07
	151	4810	"	"	"	"	"	5.65	1.738	1.100	12.70	4.35	9.62	2.92	1.32
	152	4810	"	"	"	"	"	6.65	2.046	1.350	8.20	3.24	9.57	2.53	0.86
	153	4810	"	"	"	"	"	7.65	2.354	1.600	7.20	2.58	7.60	2.79	0.95
	154	4810	"	"	"	"	"	8.65	2.662	1.850	5.70	2.13	6.38	2.67	0.89

Table No. 2 Comparison of test results with calculated values (square columns)

Source	Column Number	f'_c	f_y	b	d	p	p'	e	e/d	e'/t	P_u Test	P ACI	P_u Ultimate	Test / ACI	Test / Ultimate
	176	6670	46600	4.0	3.25	0.0302	0.0302	2.65	0.815	0.350	60.25	14.14	55.79	4.26	1.08
	177	9540	"	"	"	"	"	3.65	1.123	0.600	36.76	13.60	41.95	2.70	0.88
	178	7175	"	"	"	"	"	4.65	1.431	0.850	23.00	8.58	24.10	2.68	0.95
	179	7175	"	"	"	"	"	5.65	1.738	1.100	15.80	5.17	16.73	3.06	0.94
	180	7175	"	"	"	"	"	6.65	2.046	1.350	12.50	4.33	12.68	2.89	0.99
	181	7175	"	"	"	"	"	7.65	2.354	1.600	9.95	3.48	10.07	2.86	0.99
#12	182	7175	"	"	"	"	"	8.65	2.662	1.850	7.80	2.86	8.48	2.72	0.92
	190	4810	"	"	"	"	"	2.65	0.815	0.350	35.20	11.28	45.04	3.12	0.78
	191	4810	"	"	"	"	"	3.65	1.123	0.600	28.80	8.15	26.58	3.54	1.08
	192	4810	"	"	"	"	"	4.65	1.431	0.850	16.65	6.40	17.22	2.60	0.97
	193	5470	"	"	"	"	"	5.65	1.738	1.100	15.00	3.59	14.83	4.18	1.01
	194	5470	"	"	"	"	"	6.65	2.046	1.350	10.65	3.15	11.29	3.38	0.94
	195	5470	"	"	"	"	"	7.65	2.354	1.600	8.70	2.76	9.03	3.15	0.96
	196	6755	"	"	"	"	"	8.65	2.662	1.850	8.33	2.90	8.36	2.88	1.00

Table No. 3 Comparison of test results with calculated values (round columns)

Source	Column Number	f'_c	f_y	D	d	p_t	e'	e'/t	P_u Test	P ACI	P_u Ultimate	$\frac{\text{Test}}{\text{ACI}}$	$\frac{\text{Test}}{\text{Ultimate}}$
	A-16a	5150	43600	12	10	0.0425	0	0	760	208	703	3.65	1.08
	b	4640	"	"	"	"	0.01	0.001	770	195	652	3.94	1.18
	B-16a	2990	"	"	"	"	0.03	0.002	644	153	491	4.21	1.31
	b	3310	"	"	"	"	0.01	0.001	655	161	525	4.06	1.25
	C-16a	1590	"	"	"	"	0.03	0.002	447	117	359	3.82	1.24
	b	1420	"	"	"	"	0.02	0.002	398	113	344	3.52	1.16
	A-17a	5150	"	"	"	"	3.30	0.275	343	88.2	328	3.89	1.05
	b	4640	"	"	"	"	3.29	0.274	283	82.3	307	3.44	0.92
	B-17a	3620	"	"	"	"	3.34	0.278	253	69.6	260	3.63	0.97
	b	3310	"	"	"	"	3.34	0.278	238	65.9	247	3.61	0.96
	C-17a	1420	"	"	"	"	3.55	0.296	187	42.0	161	4.45	1.16
	b	1600	"	"	"	"	3.50	0.292	179	44.4	170	4.03	1.05
	A-18a	5020	"	"	"	"	6.44	0.536	162	55.9	190	2.90	0.85
	b	5000	"	"	"	"	6.50	0.541	171	55.4	188	3.09	0.91
#11	B-18a	3380	"	"	"	"	6.42	0.535	140	43.2	162	3.24	0.86
	b	3580	"	"	"	"	6.47	0.539	136	44.7	165	3.04	0.82
	C-18a	1680	"	"	"	"	6.80	0.567	127	28.6	115	4.44	1.10
	b	1590	"	"	"	"	6.60	0.550	107	28.6	115	3.74	0.93
	A-19a	5020	"	"	"	"	9.62	0.802	111.0	41.2	112	2.70	0.99
	b	5310	"	"	"	"	9.62	0.802	114.3	42.9	114	2.66	1.00
	B-19a	3380	"	"	"	"	9.54	0.795	98.5	31.9	105	3.08	0.94
	b	3580	"	"	"	"	9.56	0.797	103.0	33.0	106	3.12	0.97
	C-19a	1680	"	"	"	"	9.80	0.817	79.0	21.4	85	3.70	0.93
	b	1630	"	"	"	"	9.80	0.817	79.0	21.1	84	3.74	0.94
	A-20a	5310	"	"	"	"	15.68	1.307	67.7	17.9	57.2	3.78	1.18
	b	5000	"	"	"	"	15.58	1.298	63.5	17.6	57.5	3.61	1.10
	B-20a	2990	"	"	"	"	15.75	1.312	57.5	15.6	54.7	3.68	1.05
	b	3620	"	"	"	"	15.60	1.300	62.0	17.2	56.2	3.60	1.10
	C-20a	1630	"	"	"	"	15.60	1.300	47.0	13.3	51.8	3.53	0.91
	b	1600	"	"	"	"	15.72	1.310	47.0	13.2	51.3	3.56	0.92

Table No. 4 Comparison of ultimate strength indicated by three stress blocks

Source	Beam Number	$M_u/bd^2f'_c$				Test/Ultimate		
		Test	Parabolic	Trapezoidal	Rectangular	Parabolic	Trapezoidal	Rectangular
#1	1	0.506	0.332	0.363	0.333	1.52	1.39	1.52
	2	0.337	0.318	0.343	"	1.06	0.98	1.01
	3	0.326	0.311	0.338	"	1.05	0.96	0.98
	4	0.339	0.314	0.342	"	1.08	0.99	1.02
	5	0.320	0.314	0.342	"	1.02	0.94	0.96
	6	0.422	0.322	0.351	"	1.31	1.20	1.27
	6A	0.327	0.312	0.340	"	1.05	0.96	0.98
	7	0.341	0.312	0.340	"	1.09	1.00	1.02
	8	0.380	0.320	0.349	"	1.19	1.09	1.14
	9	0.391	0.320	0.349	"	1.22	1.12	1.17
	10	0.346	0.318	0.347	"	1.09	1.00	1.04
	10A	0.354	0.317	0.346	"	1.12	1.02	1.06
#6	C1	0.406	0.313	0.341	"	1.30	1.19	1.22
	C11	0.394	0.313	0.341	"	1.26	1.15	1.18
	C2	0.386	0.312	0.340	"	1.24	1.14	1.16
	C12	0.365	0.311	0.339	"	1.17	1.08	1.10
#7	122	0.405	0.317	0.345	"	1.28	1.17	1.22
	123	0.423	0.333	0.365	"	1.27	1.16	1.27
	124	0.455	0.343	0.376	"	1.33	1.21	1.37
	125	0.471	0.349	0.384	"	1.35	1.23	1.41
	223	0.350	0.307	0.334	"	1.14	1.05	1.05
	224	0.343	0.321	0.350	"	1.07	0.98	1.03
	225	0.353	0.330	0.360	"	1.07	0.98	1.06
	243	0.370	0.322	0.352	"	1.15	1.05	1.11
	244	0.375	0.333	0.365	"	1.13	1.03	1.13
	214	0.374	0.315	0.343	"	1.19	1.09	1.12
	215	0.372	0.325	0.351	"	1.14	1.06	1.07

Table No. 4 Comparison of ultimate strength indicated by three stress blocks

Source	Beam Number	$M_u/bd^2f'_c$				Test/Ultimate		
		Test	Parabolic	Trapezoidal	Rectangular	Parabolic	Trapezoidal	Rectangular
#7	234	0.348	0.315	0.343	0.333	1.10	1.01	1.04
	235	0.381	0.325	0.355	"	1.17	1.07	1.14
	252	0.379	0.320	0.349	"	1.18	1.09	1.14
	253	0.388	0.336	0.368	"	1.15	1.05	1.17
	324	0.294	0.303	0.329	0.314	0.97	0.89	0.94
	325	0.320	0.314	0.342	0.333	1.02	0.94	0.96
	344	0.342	0.319	0.348	"	1.07	0.98	1.03
#8	4203	0.390	0.323	0.352	"	1.21	1.11	1.17
	4204	0.463	0.333	0.365	"	1.39	1.27	1.39
	4205	0.449	0.342	0.375	"	1.31	1.20	1.35
	4206	0.464	0.347	0.382	"	1.34	1.21	1.39
	4305	0.382	0.320	0.349	"	1.19	1.09	1.15
	4306	0.351	0.319	0.348	0.326	1.10	1.01	1.08
	4307	0.440	0.330	0.361	0.333	1.33	1.22	1.32
	4308	0.367	0.328	0.359	"	1.12	1.02	1.10
	4408	0.346	0.317	0.346	"	1.09	1.00	1.04
	6203	0.347	0.293	0.318	"	1.18	1.09	1.04
	6204	0.333	0.306	0.339	"	1.09	0.98	1.00
	6205	0.415	0.322	0.351	"	1.29	1.18	1.25
	6206	0.400	0.328	0.359	"	1.22	1.11	1.20
	6207	0.391	0.342	0.375	"	1.14	1.04	1.17
	6208	0.407	0.339	0.372	"	1.20	1.09	1.22
	6304	0.347	0.307	0.334	"	1.13	1.04	1.04
	6305	0.338	0.310	0.337	"	1.09	1.00	1.01
	6306	0.359	0.331	0.361	"	1.08	0.99	1.08
	6405	0.293	0.295	0.321	"	0.99	0.91	0.88
	6406	0.317	0.312	0.339	"	1.02	0.94	0.95
	6407	0.314	0.313	0.342	"	1.00	0.92	0.94
	6505	0.247	0.306	0.333	"	0.81	0.74	0.74
	6506	0.292	0.307	0.334	"	0.95	0.87	0.88

Table No. 4 Comparison of ultimate strength indicated by three stress blocks

Source	Beam Number	$M_u/bd^2f'_c$				Test/Ultimate		
		Test	Parabolic	Trapezoidal	Rectangular	Parabolic	Trapezoidal	Rectangular
#9	1110	0.347	0.328	0.362	0.333	1.06	0.96	1.04
	1112	0.414	0.329	0.359	"	1.26	1.15	1.24
	1113	0.316	0.331	0.362	"	0.94	0.87	0.95
	1116	0.364	0.310	0.310	0.310	1.17	1.17	1.17
	1119	0.281	0.336	0.336	0.333	0.84	0.84	0.84
	1120	0.280	0.326	0.346	"	0.86	0.81	0.84
	1121	0.283	0.331	0.361	"	0.85	0.78	0.85
	1123	0.351	0.328	0.358	"	1.07	0.98	1.05
	1125	0.367	0.332	0.363	"	1.10	1.01	1.10
	1126	0.327	0.330	0.362	"	0.99	0.90	0.98
	1128	0.368	0.326	0.350	"	1.13	1.05	1.10
	1129	0.344	0.334	0.334	"	1.03	1.03	1.03
	1130	0.341	0.324	0.341	"	1.05	1.00	1.02
	1131	0.265	0.318	0.318	0.318	0.83	0.83	0.83
	1132	0.285	0.321	0.321	0.321	0.89	0.89	0.89
	1133	0.286	0.319	0.319	0.319	0.90	0.90	0.90
	1135	0.310	0.314	0.314	0.314	0.99	0.99	0.99
	1137	0.296	0.326	0.326	0.326	0.91	0.91	0.91
	1138	0.318	0.337	0.337	0.333	0.94	0.94	0.95
	1139	0.352	0.328	0.359	0.333	1.07	0.98	1.06

APPENDIX B

Design Aids:

To expedite the proportioning of sections by ultimate strength theory, a series of charts has been prepared. Fig. 5 enables a designer to determine either the percentage of reinforcement required after a section has been chosen or the depth of beam for a given percentage of reinforcement and width of beam. In the first case, the chart is entered from the left side with a calculated value of M_u/bd^2. This value is extended to the cylinder strength chosen. The intersection is then extended vertically upwards to the steel strength used. The percentage of reinforcement is read at the right hand margin. For determining the depth required for a given moment, the direction is reversed.

There are several ways of using the design chart in Fig. 6. One way is to enter chart with a value of $P_u/f'_c bd$ and proceed horizontally to a value of $pm - p'm$ which for symmetrical reinforcement equals 0. From this intersection proceed vertically to the calculated value of $P_u e/f'_c bd^2$ and then horizontally to the right margin and read the value of $p'm\ (1-d'/d)$. From this value the amount of compression and tensile reinforcement can be obtained.

The next four charts of Fig. 7 are applicable to symmetrically reinforced rectangular columns. The intersection of the values of $P_u/f'_c bt$ and $P_u e'/f'_c bt^2$ gives the required amount of reinforcement.

APPENDIX C

Derivations

The derivation of ultimate strength formulas for reinforced concrete cross sections adheres closely to the procedure employed in the straight line theory. As in the classical approach, the determination of the capacity of a section commences with the two basic equations of equilibrium.

Rectangular Beams in Simple Bending with No Compressive Reinforcement

In Fig. 8, the summation of horizontal forces at ultimate load yields

$$0 = 0.85 f'_c k_1 k_u bd - A_s f_s \tag{17}$$

and the summation of moments about the reinforcement gives

$$M_u = 0.85 f'_c k_1 k_u (1 - k_2 k_u)\, bd^2 \tag{18}$$

in which

f'_c = 28 day cylinder strength of the concrete
k_1 = the average stress in the compressive area divided by 0.85 f'_c
$k_u d$ = distance from extreme compressive fiber to the neutral axis
b = width of section
d = depth of beam to centroid of reinforcement
A_s = area of tensile reinforcement
f_s = tension in reinforcement
$k_2 k_u d$ = distance from extreme fiber to centroid of compressive force

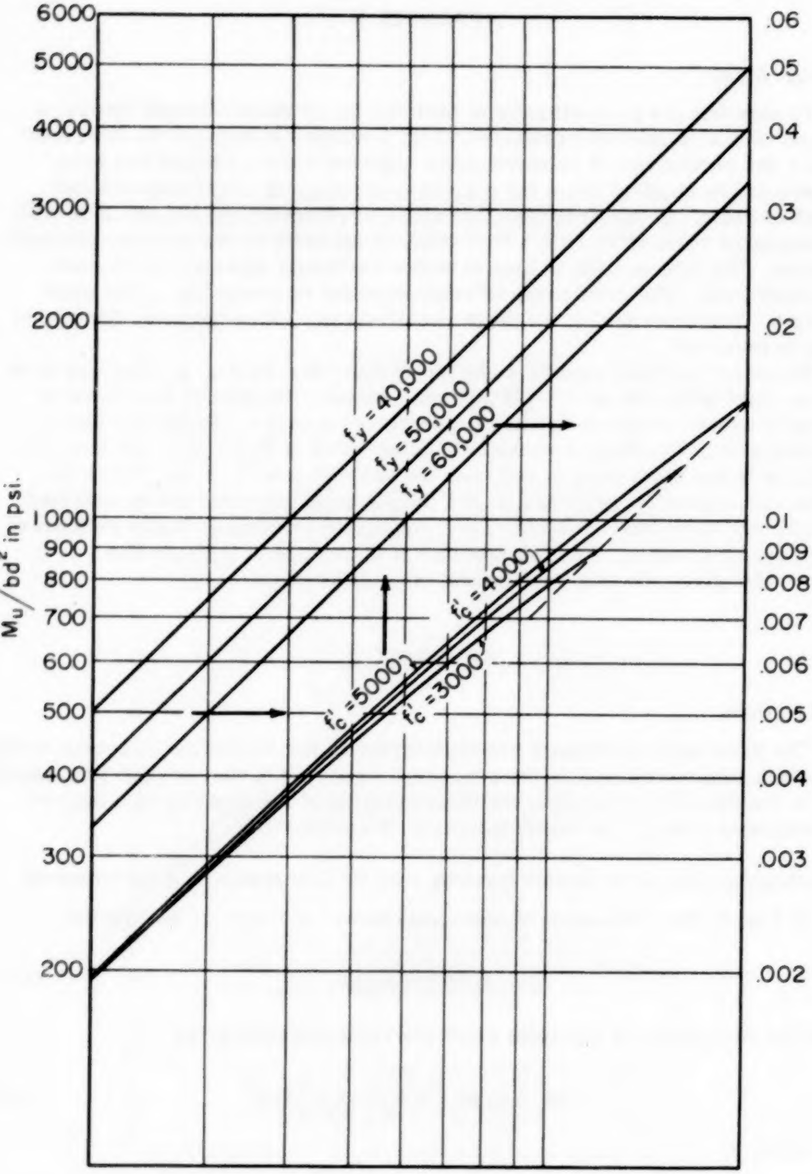

Fig. 5. Ultimate Bending Moment for Rectangular Beams.

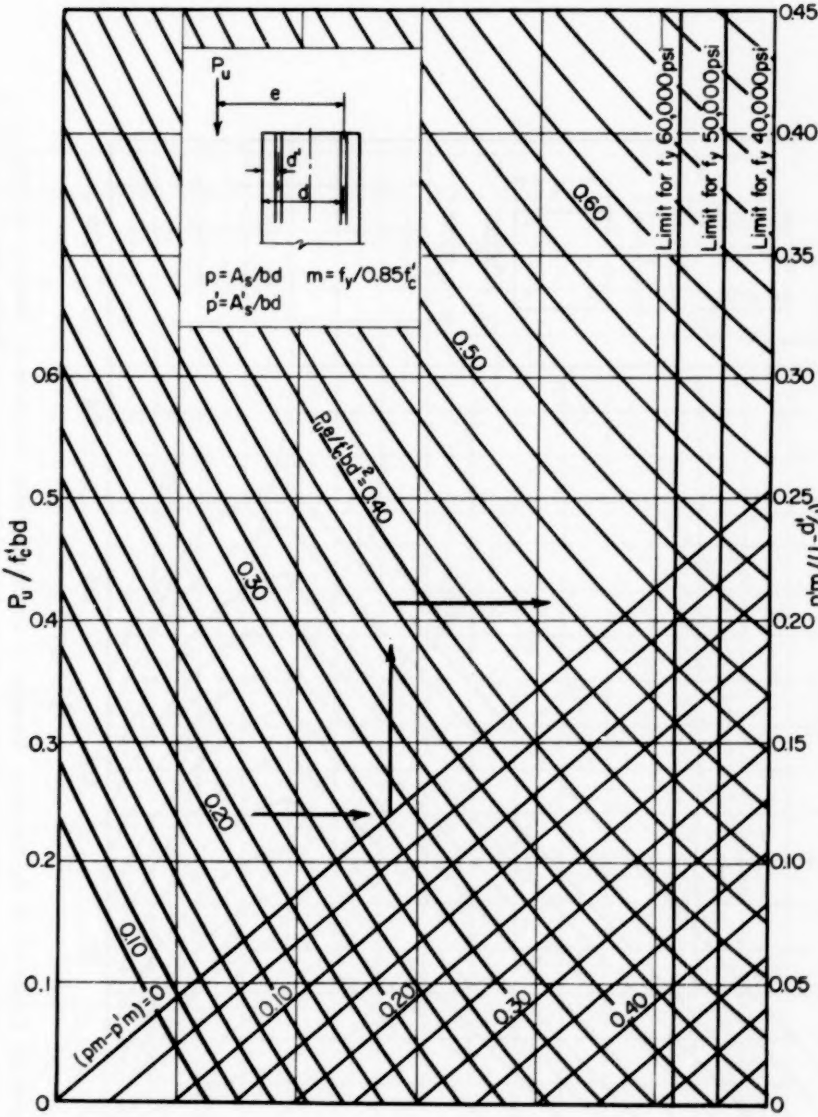

Fig. 6. Ultimate Capacity for Rectangular Members Subject to Combined Bending and Axial Load, and Controlled by Tension.

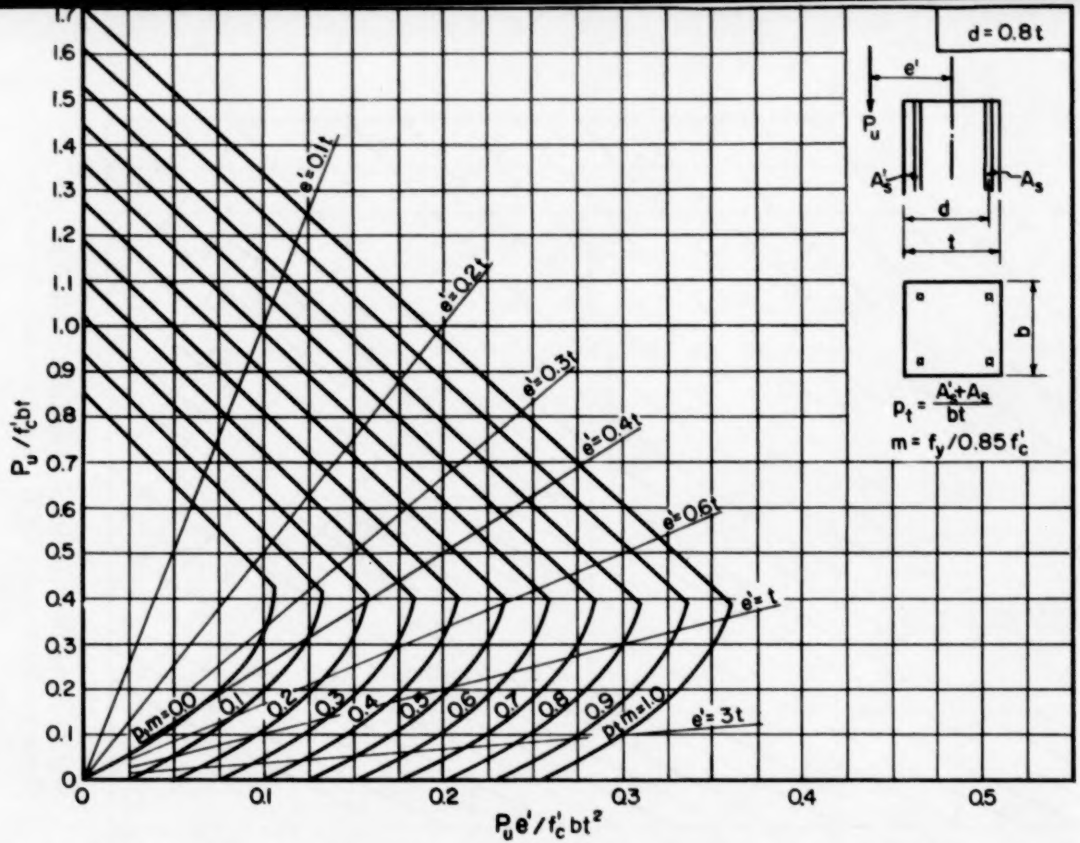

Fig. 7. Ultimate Capacity of Rectangular Members, Symmetrically Reinforced, Subject to Combined Bending and Axial Load (From Journal of Boston Society of Civil Engineers, Vol. XXXV, No. 1, Jan. 1948).

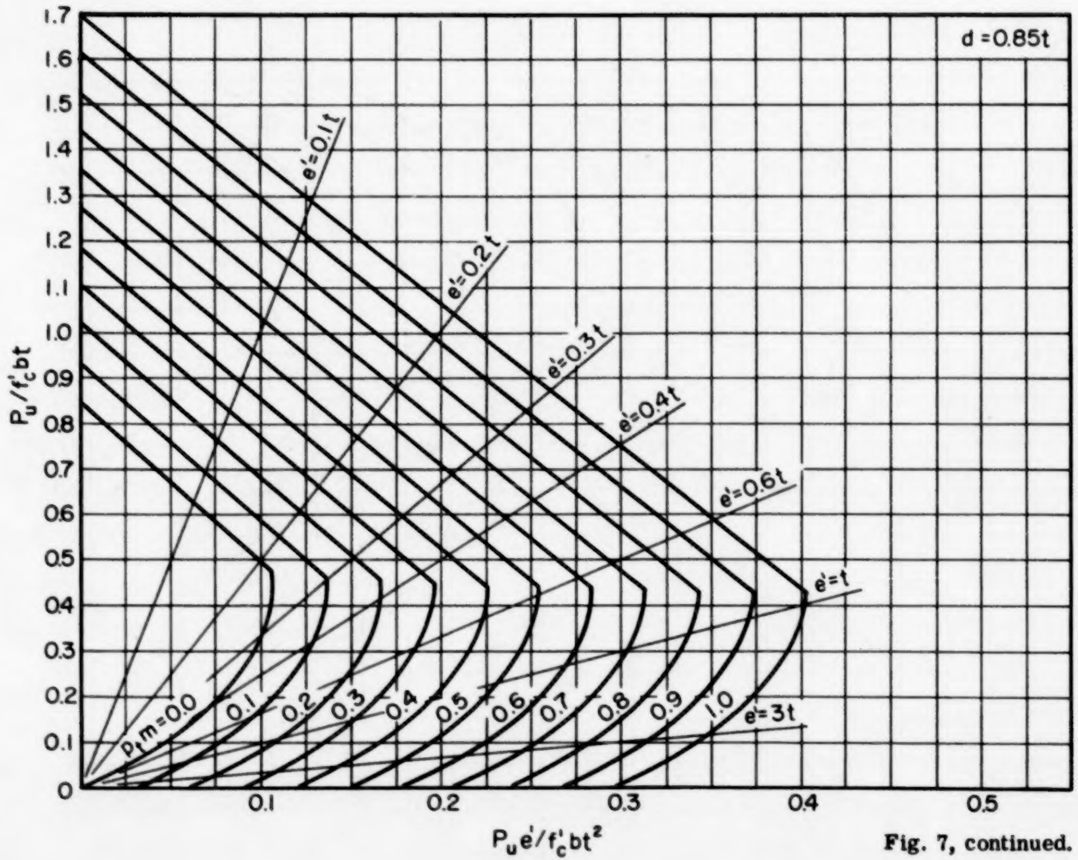

Fig. 7, continued.

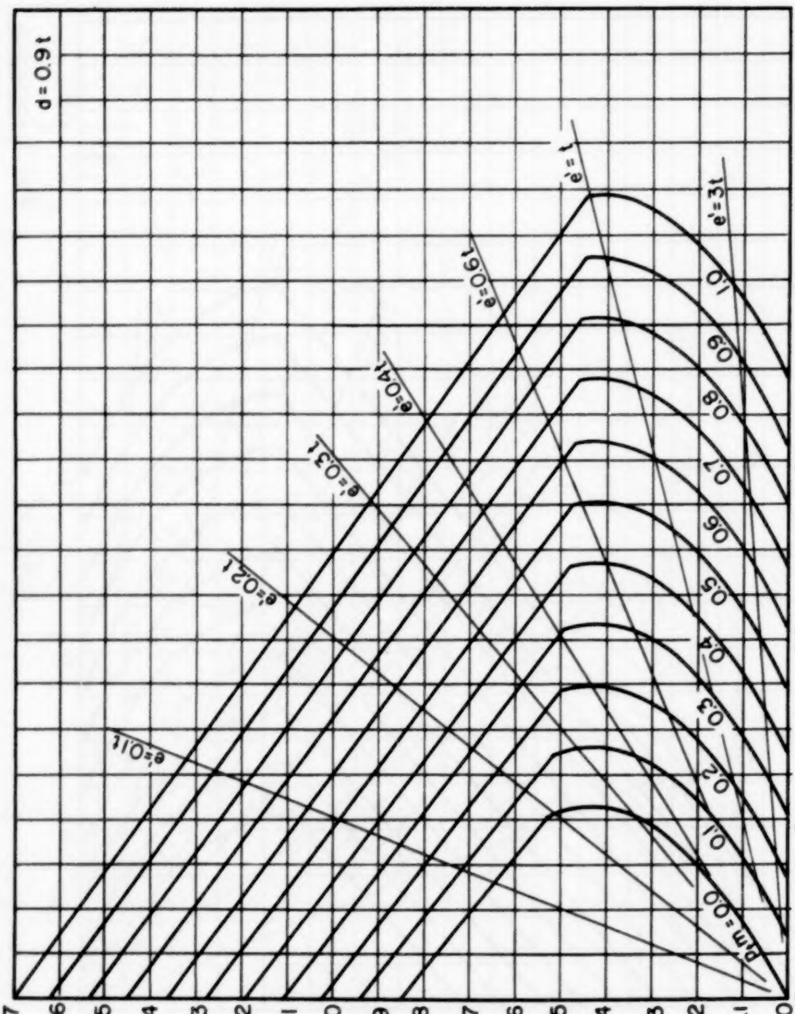

Fig. 7, continued.

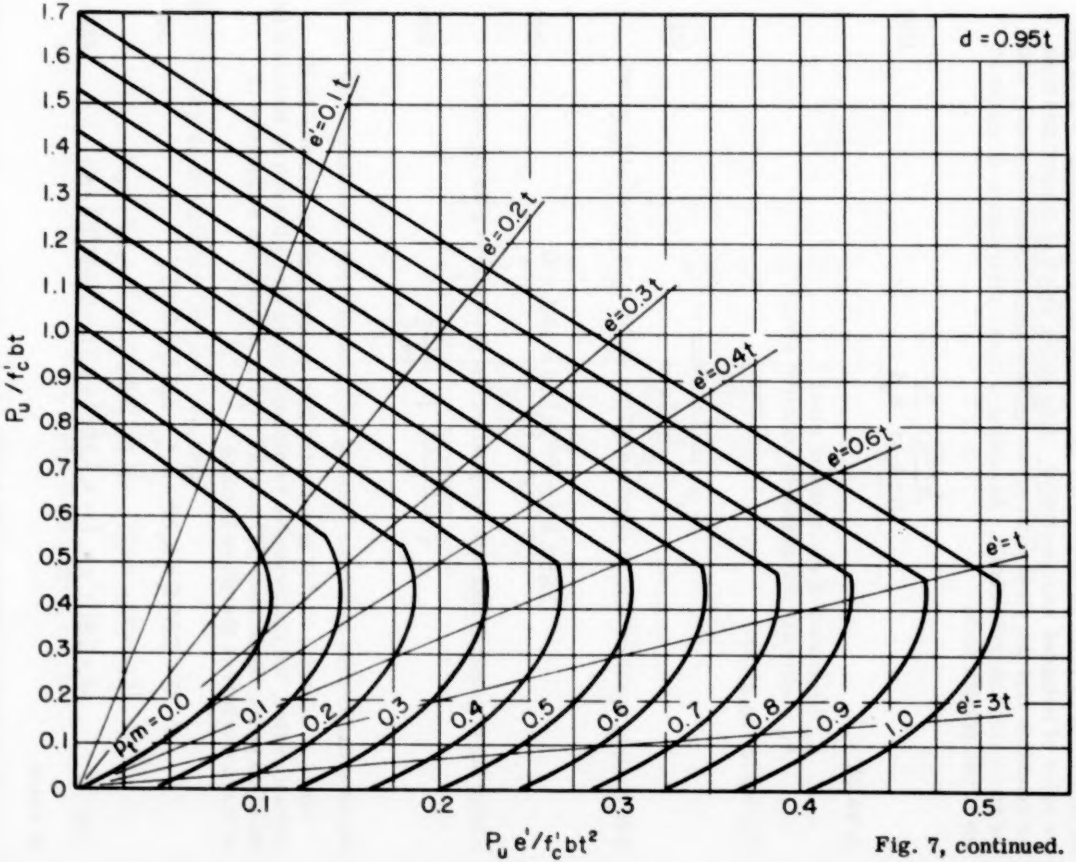

Fig. 7, continued.

Combining equation (17) and (18) and assuming that $f_s = f_y$, we have

$$M_u = A_s f_y d\left(1 - \frac{k_2}{k_1} \times pm\right) \tag{19}$$

in which f_y = yield point stress of steel
$p = A_s/bd$
$m = f_y/0.85\, f'_c$

Equation (19) is valid as long as the percentage of reinforcement is less than the value of balanced reinforcement. This latter value is determined from the condition that the yield point strain in steel occurs simultaneously with the maximum concrete strain. Assuming a linear distribution of strain at any cross section then,

$$\frac{f_y/E_s}{(1-k_u)d} = \frac{\epsilon_u}{k_u d} \tag{20}$$

in which

E_s = modulus of elasticity of steel
ϵ_u = ultimate compressive concrete strain

Solving for k_u gives

$$k_u = \frac{\epsilon_u E_s}{f_y + \epsilon_u E_s} \tag{21}$$

Substituting equation (21) in equation (17) in which f_s is taken at f_y gives

$$0.85\, f'_c k_1 \left(\frac{\epsilon_u E_s}{f_y + \epsilon_u E_s}\right) bd - A_s f_y = 0 \tag{22}$$

Factoring common terms and designating the determined p as p_b gives

$$p_b = \frac{0.85\, f'_c k_1}{f_y} \left(\frac{\epsilon_u E_s}{f_y + \epsilon_u E_s}\right) \tag{23}$$

Rectangular Beams in Simple Bending with Compressive Reinforcement

Since at ultimate load it is assumed that the concrete is strained to its ultimate value, the compressive reinforcement is also considered stressed to its yield point. Equations (17) and (18) which express the summation of moment and forces therefore become

$$0 = 0.85\, f'_c k_1 k_u bd - A_s f_s + A'_s (f_y - 0.85\, f'_c) \tag{24}$$

$$M_u = 0.85\, f'_c k_1 k_u (1 - k_2 k_u) bd^2 + A'_s (f_y - 0.85\, f'_c)(d - d') \tag{25}$$

in which

A'_s = area of compressive reinforcement
d' = distance from extreme fiber to centroid of compression reinforcement

Assuming $f_s = f_y$ in equation (24) and solving for k_u

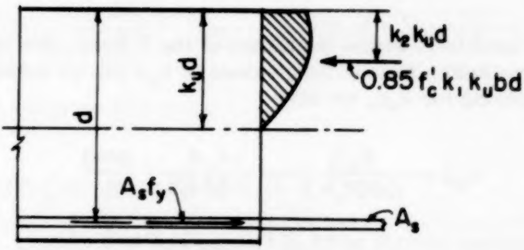

Fig. 8.

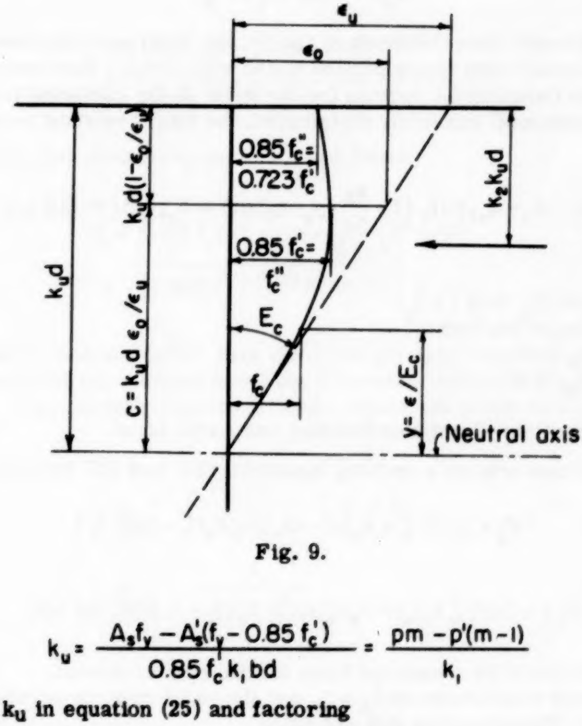

Fig. 9.

$$k_u = \frac{A_s f_y - A'_s(f_y - 0.85 f'_c)}{0.85 f_c k_1 bd} = \frac{pm - p'(m-1)}{k_1} \tag{26}$$

Substituting k_u in equation (25) and factoring

$$M_u = \left[A_s f_y - A'_s(f_y - 0.85 f'_c)\right] d \left[1 - \frac{k_2}{k_1}\left(pm - p'(m-1)\right)\right] + A'_s(f_y - 0.85 f'_c)(d - d') \tag{27}$$

Because of their minor effects, the term $0.85\ f'_c$ in the first parenthesis and -1 in the quantity multiplying p can be ignored. Hence

$$M_u = (A_s - A'_s) f_y d \left[1 - \frac{k_2}{k_1}(p - p')m\right] + A'_s f_y (d - d') \tag{28}$$

T-beams

If the neutral axis falls within the flange of the T beam, the formulas for rectangular beams hold. The limiting values of $k_u d$ can be estimated from equation (17). Solving for $k_u d$, we have

$$k_u d = \frac{A_s f_y}{0.85 f'_c k_1 b} = \frac{p f_y d}{0.85 f'_c k_1} = \frac{pmd}{k_1} \tag{29a}$$

in which p is based on overall width of flanges and web or b.

If k_1 is assumed conservatively as 0.9, then

$$k_u d = 1.30\, p f_y d / f'_c \tag{29b}$$

When $k_u d$ is greater than the depth of flange, the steel provided may be considered to be subdivided into a portion which will develop the compressive strength of the flange and a portion for the web. If the compressive force in the flange is assumed uniformly distributed, the total resisting moment is

$$M_u = (A_s - A_{sf}) d f_y \left(1 - \frac{k_2}{k_1}(p_w - p_f) m\right) + A_{sf} f_y d (1 - 0.5 t/d) \tag{30}$$

in which

$A_{sf} = 0.85 f'_c (b-b') t/f_y$
t = flange thickness
$p_w = A_s/b'd$
$p_f = A_{sf}/b'd$

Rectangular Section: Combined Bending and Axial Load

If an axial load acts on a section, equations (24) and (25) become

$$P_u = 0.85 f'_c k_1 k_u bd - A_s f_s + A'_s (f_y - 0.85 f'_c) \tag{31}$$

$$P_u e = 0.85 f'_c k_1 k_u (1 - k_2 k_u) bd^2 + A'_s (f_y - 0.85 f'_c)(d - d') \tag{32}$$

with e, the eccentricity measured from tensile reinforcement.

For balanced reinforcement $f_s = f_y$ and the value of k_u is expressed by equation (21). Therefore for this condition

$$P_b = 0.85 f'_c k_1 \left(\frac{\epsilon_u E_s}{f_y + \epsilon_u E_s}\right) bd - A_s f_y + A'_s (f_y - 0.85 f'_c) \tag{33}$$

Whenever the axial load exceeds the value given by the right side of equation (33), compression strength controls and when the axial load is less, tension strength controls.

For tension failure, with $f_s = f_y$, dividing equation (32) by equation (31) and factoring

$$k_u = \frac{(1-e/d)}{2k_2} + \sqrt{\left(\frac{1-e/d}{2k_2}\right)^2 + \frac{[pm - p'(m-1)]e/d + p'(m-1)(1-d'/d)}{k_1 k_2}} \tag{34}$$

Substituting k_u in equation (31) gives

$$P_u = 0.85 f_c' bd \left\{ (p'-p)\, m - p' + \frac{k_1}{2k_2}\left[(1-e/d) + \sqrt{(1-e/d)^2 + \frac{4k_2}{k_1}\left[(pm - p'(m-1))\, e/d + p'(m-1)(1-d'/d)\right]}\right]\right\} \tag{35}$$

For symmetrical reinforcement p = p', and therefore

$$P_u = 0.85 f_c' bd \left\{ -p' + \frac{k_1}{2k_2}\left[(1-e/d) + \sqrt{(1-e/d)^2 + \frac{4k_2 p}{k_1}\left(e/d + (m-1)(1-d'/d)\right)}\right]\right\} \tag{36a}$$

With no compression reinforcement p' = o and hence

$$P_u = 0.85 f_c' bd \left\{ -pm + \frac{k_1}{2k_2}\left[(1-e/d) + \sqrt{(1-e/d)^2 + \frac{4k_2}{k_1} pm\, e/d}\right]\right\} \tag{36b}$$

When the axial load is greater than the value given by equation (33), the relationship between the moment about the centroidal axis and the thrust is almost linear. For design purposes a linear relationship can be assumed. On this basis:

$$P_u = \left(\frac{P_o - P_b}{-P_b e_b'}\right) P_u e' + P_o \tag{37a}$$

or

$$P_u = \frac{P_o}{1 + \frac{e'}{e_b'}\left(\frac{P_o}{P_b} - 1\right)} \tag{37b}$$

in which

p_o = concentric load capacity
e' = eccentricity measured from plastic centroid of section
e_b' = eccentricity of load P_b measured from plastic centroid of section

For a concentrically loaded specimen

$$P_o = 0.85 f_c (A_g - A_s) + A_s f_y \tag{38}$$

in which

A_g = the gross area of section
A_s = cross-sectional area of bars

The preceding equations represent generalized statements valid for any type of stress distribution. To obtain numerical values for the constants k_1 and k_2, a particular stress block distribution must be assumed. For the purpose of studying the effect of changes in the variables on the ultimate capacity, three representative distributions, parabolic, trapezoidal and rectangular will be investigated.

Parabolic Distribution

When the initial stress-strain relationship is assumed equal to E_c, the parabolic distribution must conform to a shape as shown in Fig. 9, in which the solid outline represents stress relationship and the dash line, strain relationship. In this figure, the curve between the neutral axis and the ordinate $\epsilon_o k_u d/\epsilon_u$ is assumed to be a second degree parabola. Above this ordinate a linear variation is assumed. Thus the area in the upper portion of the stress block is trapezoidal. On this basis, the stress at any point in terms of the distance from the neutral axis is

$$f_c'' - f_c = \frac{f_c''}{(c)^2}(c-y)^2 \qquad (39a)$$

But since the strain is assumed to vary linearly from top to bottom

$$y = \epsilon/E_c \qquad (39b)$$

and

$$c = \epsilon_o/E_c \qquad (39c)$$

Substituting these values in equation (39a) gives

$$f_c'' - f_c = \frac{f_c''}{\epsilon_o^2}(\epsilon_o - \epsilon)^2 \qquad (39d)$$

From which

$$f_c = f_c''\left[2\epsilon/\epsilon_o - (\epsilon/\epsilon_o)^2\right] \qquad (39e)$$

The slope of this curve represents the variation in the modulus of elasticity. Hence taking the first derivative, gives

$$\frac{df_c}{d\epsilon} = f_c''\left[2/\epsilon_o - 2\epsilon/(\epsilon_o)^2\right] \qquad (39f)$$

At the neutral axis, it is assumed that the slope is equal to E_c, and therefore when $\epsilon = o$, we have

$$E_c = \frac{2f_c''}{\epsilon_o} \qquad (39g)$$

or

$$\epsilon_o = \frac{2f_c''}{E_c} = 1.7\, f_c' / E_c \tag{39h}$$

From Fig. 9, the area of the compressive block being equal to the sum of the parabolic and trapezoidal segments is equal to

$$0.85 f_c' \times \frac{2bc}{3} + \frac{(0.723+0.85)}{2} f_c' b\,(k_u d - c) \tag{40a}$$

$$f_c' b k_u d \left[0.567 \epsilon_o/\epsilon_u + 0.786(1-\epsilon_o/\epsilon_u)\right] \tag{40b}$$

$$f_c' b k_u d \left[0.786 - 0.219\, \epsilon_o/\epsilon_u\right] \tag{40c}$$

Since by definition k_1 is the ratio of the average stress to 0.85 f'_c

$$k_1 = \frac{0.786 - 0.219\, \epsilon_o/\epsilon_u}{0.85} \tag{40d}$$

$$= 0.925 - 0.258\, \epsilon_o/\epsilon_u \tag{40e}$$

Substituting for ϵ_o its value given by equation (39h)

$$k_1 = 0.925 - 0.438\, f_c'/E_c \epsilon_u \tag{41}$$

The value of k_2 is determined by taking moments of the compressive force about the top of beam. The lever arm from top of beam to centroid of parabolic section is

$$k_u d(1-\epsilon_o/\epsilon_u) + \tfrac{3}{8} k_u d \epsilon_o/\epsilon_u = k_u d(1 - 0.625\, \epsilon_o/\epsilon_u) \tag{42a}$$

and that for the trapezoidal portion is

$$\frac{k_u d}{3}(1-\epsilon_o/\epsilon_u)\left(1 + \frac{0.85}{0.85+0.723}\right) = 0.513\, k_u d(1-\epsilon_o/\epsilon_u) \tag{42b}$$

Multiplying the areas by the moment arms we have for the moment

$$f_c' b(k_u d)^2 \left[(1-0.625\, \epsilon_o/\epsilon_u)\, 0.567\, \epsilon_o/\epsilon_u + 0.513(1-\epsilon_o/\epsilon_u)^2\, 0.786\right] \tag{42c}$$

Factoring

$$f_c' b(k_u d)^2 \left[0.403 - 0.239\, \epsilon_o/\epsilon_u + 0.0488\, (\epsilon_o/\epsilon_u)^2\right] \tag{42d}$$

Dividing by the total area

$$k_2 k_u d = k_u d \frac{0.403 - 0.239\,\epsilon_o/\epsilon_u + 0.0488\,(\epsilon_o/\epsilon_u)^2}{0.786 - 0.219\,\epsilon_o/\epsilon_u} \tag{42e}$$

and therefore

$$k_2 = \frac{0.403 - 0.239\,\epsilon_o/\epsilon_u + 0.0488\,(\epsilon_o/\epsilon_u)^2}{0.786 - 0.219\,\epsilon_o/\epsilon_u} \tag{42f}$$

and replacing ϵ_o by its equivalent value 1.7 f'_c/E_c and dividing by k_1, gives

$$\frac{k_2}{k_1} = \frac{0.403 - 0.406 f'_c/E_c\epsilon_u + 0.142\,(f'_c/E_c\epsilon_u)^2}{\dfrac{(0.786 - 0.372 f'_c/E_c\epsilon_u)^2}{0.85}} \tag{42g}$$

which can be restated as:

$$\frac{k_2}{k_1} = 0.554\left[\frac{1 - 1.007 f'_c/E_c\epsilon_u + 0.351\,(f'_c/E_c\epsilon_u)^2}{1 - 0.946 f'_c/E_c\epsilon_u + 0.224\,(f'_c/E_c\epsilon_u)^2}\right] \tag{42h}$$

From the above equation, it is evident by comparing the numerator and denominator that k_2/k_1 is insensitive to variations in the value selected for E_c and ϵ_u. For all practical purposes

$$k_2/k_1 = 0.554 \tag{43}$$

The value of k_1 on the other hand from an inspection of eq. (41) is somewhat dependent on the selected values of ϵ_u and E_c. The maximum variation in the value of k_1 occurs at the lower cylinder strengths. Even then within the practical ranges for ϵ_u from .003 to .004, the maximum variation in k_1 is only about 9%. If ϵ_u is assumed equal to .0035 and E_c = 1000 f'_c

$$k_1 = 0.925 - \frac{0.438}{0.0035 \times 1000} = 0.80 \tag{44}$$

Trapezoidal Distribution

In Fig. 10, the solid outline represents the distribution of compressive stresses while the dash line represents the strain. From geometrical considerations the total compressive area equals

$$0.85 f'_c b k_u d\, \frac{\left[1 + (1 - \epsilon_o/\epsilon_u)\right]}{2} \tag{45a}$$

$$0.85 f'_c b k_u d (1 - \epsilon_o/2\epsilon_u) \tag{45b}$$

and therefore

$$k_1 = 1 - \epsilon_o/2\epsilon_u \tag{46}$$

The moment of the compressive forces about the top of the beam is

$$\frac{0.85 f'_c\, b(k_u d)^2 \epsilon_o/\epsilon_u(1-0.67\epsilon_o/\epsilon_u)}{2} + \frac{0.85 f'_c\, b(k_u d)^2(1-\epsilon_o/\epsilon_u)^2}{2} \tag{47}$$

Factoring gives

$$0.425 f'_c\, b(k_u d)^2\left[1-\epsilon_o/\epsilon_u + 0.33(\epsilon_o/\epsilon_u)^2\right] \tag{47b}$$

Dividing by the area and recalling that by definition $k_2 k_u d$ is the distance from top of beam to centroid of force

$$k_2 = \frac{0.5\left[1-\epsilon_o/\epsilon_u + 0.33(\epsilon_o/\epsilon_u)^2\right]}{1-\epsilon_o/2\epsilon_u} \tag{47c}$$

and hence

$$k_2/k_1 = 0.5\,\frac{1-\epsilon_o/\epsilon_u + 0.33(\epsilon_o/\epsilon_u)^2}{(1-\epsilon_o/2\epsilon_u)^2} \tag{47d}$$

Now

$$\epsilon_o = 0.85 f'_c/E_c \tag{48}$$

Hence equation (46) reduces to

$$k_1 = 1 - 0.425 f'_c/E_c\epsilon_u \tag{49}$$

and

$$k_2/k_1 = 0.5\left[\frac{1-0.85 f'_c/E_c\epsilon_u + 0.241(f'_c/E_c\epsilon_u)^2}{1-0.85 f'_c/E_c\epsilon_u + 0.181(f'_c/E_c\epsilon_u)^2}\right] \tag{50}$$

For the same reason as given in parabolic distribution, for practical purposes

$$k_2/k_1 = 0.50 \tag{51}$$

Following the same procedure as used in the parabolic stress block, variation in the value of k_1 may amount to as much as 8%. If ϵ_u is assumed equal to .0035 and $E_c = 1000 f'_c$

$$k_1 = 1 - \frac{0.425}{3.5} = 0.879 \tag{52}$$

Rectangular Distribution

From the geometrical properties of a rectangle, for the rectangular

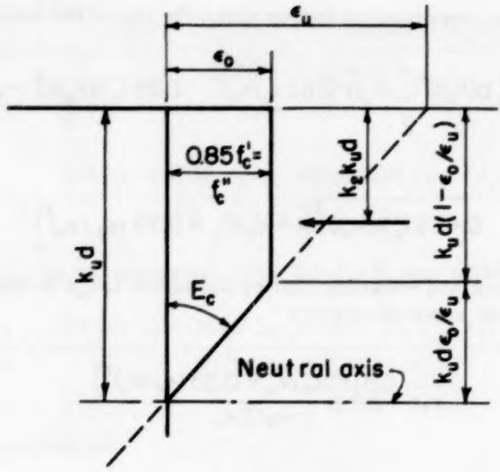

Fig. 10.

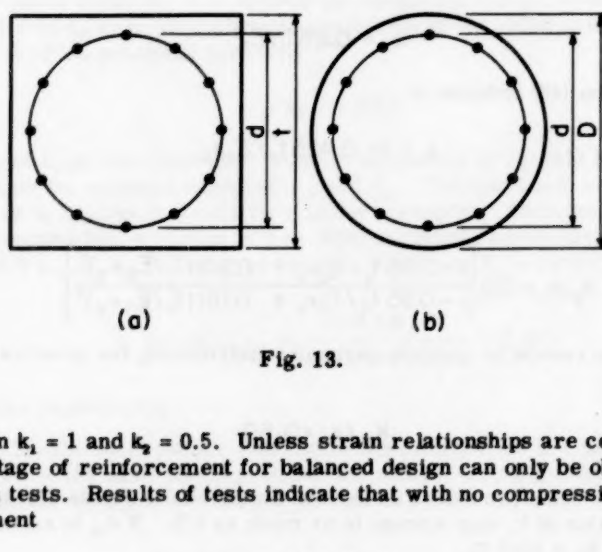

Fig. 13.

distribution $k_1 = 1$ and $k_2 = 0.5$. Unless strain relationships are considered, the percentage of reinforcement for balanced design can only be obtained from beam tests. Results of tests indicate that with no compression reinforcement

$$p_b = 0.456\, f_c'/f_y \tag{53}$$

is the critical percentage of steel required to develop the full compressive strength of the concrete.

The preceding derivations have been formulated on the basis of a non-linear stress-strain relationship. However for the rectangular distribution, another approach only indirectly related to the stress-strain relationship has the merit of considerable simplicity.

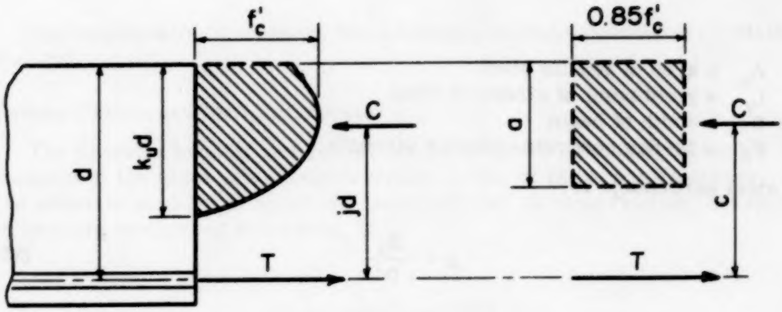

Fig. 11.

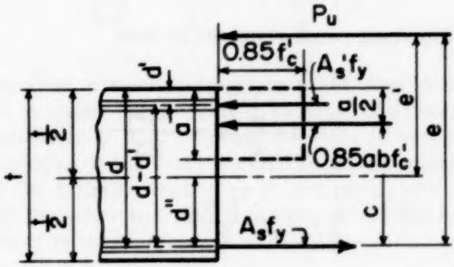

Fig. 12.

If it is assumed that the stress distribution in a concrete beam at failure has the shape of the cylinder stress-strain curve shown in Fig. 11, the total compression, C, is the area bounded by the curve. The line of action of C lies through the center of gravity of this area. From an examination of actual stress-strain curves it is found that, if the actual area is replaced by an equivalent rectangular area of width equal to $0.85f'_c$ and depth equal to a, the location of the center of gravity of this rectangle corresponds closely with that of the actual area.

If the beam is under-reinforced so that the primary failure will occur in the tensile steel, the concrete will crack as the steel stretches, and the equivalent depth of the beam in compression, a, will decrease until the average effective concrete stress reaches the maximum. The concrete will then fail progressively, reducing the lever arm of the steel and causing failure of the beam.

Therefore

$$a = \frac{A_s f_y}{0.85bf'_c} = \frac{A_s m}{b} \tag{54a}$$

or

$$\frac{a}{d} = pm \tag{54b}$$

in which

A_s = area of tensile steel
f_y = yield point of stress of steel
b = width of beam
f'_c = 28-day concrete cylinder strength.

The steel percentage is

$$p = \frac{A_s}{bd} \tag{55a}$$

and

$$m = \frac{f_y}{0.85 f'_c} \tag{55b}$$

The lever arm of the steel reinforcement is then

$$c = d - \frac{a}{2} = d - \frac{A_s m}{2b} \tag{56a}$$

or

$$\frac{c}{d} = 1 - \frac{pm}{2} \tag{56b}$$

The ultimate resisting moment of the beam as controlled by tension in the steel can be written:

$$M_u = c\, A_s f_y = A_s f_y \left(d - \frac{A_s m}{2b}\right) \tag{57a}$$

or

$$\frac{M_u}{bd^2} = p f_y \left(1 - \frac{pm}{2}\right) \tag{57b}$$

The limiting value of the depth of equivalent compression, a, for equal concrete and steel strengths in flexure can best be determined experimentally. If there is at least sufficient steel to fully develop the strength of the concrete, additional steel does not materially increase the strength of the beam.

The limiting value of a/d computed from tests is 0.537 and c/d is 0.732. The corresponding ultimate resisting moment is given by the expression

$$\frac{M_u}{bd^2} = 0.85 f'_c \frac{a}{d}\left(1 - \frac{a}{2d}\right) = \frac{f'_c}{3} \tag{58}$$

The critical percentage of steel required to develop the full compressive strength of the concrete is (see Eq. 54):

$$p_b = \frac{0.537}{m} = 0.456 \frac{f'_c}{f_y} \tag{59}$$

For beams with less steel, the allowable bending moment is given by Eq. (57).

Beams Reinforced for Compression

The ultimate bending moment in compression is computed by adding the moment of the steel compressive stress to that of the concrete stress. Then the ultimate moment at which the beam will fail in compression, assuming it to be fully reinforced in tension, is

$$M_u = \frac{1}{3} b d^2 f'_c + (d-d')\ A'_s f_y \tag{60a}$$

or

$$\frac{M_u}{bd^2} = \frac{f'_c}{3} + (1 - d'/d)\ p' f_y \tag{60b}$$

in which A'_S is the area of the compression steel, and (d-d') is its lever arm.

When the beam is under-reinforced in tension, the lever arm of the tensile steel can be computed as if the compressive steel is stressed to its elastic limit and the remainder of the compression is on the concrete. The calculation is made as if the tensile steel were in two parts, one to balance the compressive steel and the other to balance the stress on the concrete. The ultimate tensile moment is then

$$M_u = A'_s f_y (d-d') + (A_s - A'_s)\left[d - \frac{(A_s - A'_s)m}{2b}\right] f_y \tag{61}$$

or

$$\frac{M_u}{bd^2} = \frac{(d-d')}{d} p' f_y + (p-p')\left[1 - \frac{(p-p')m}{2}\right] f_y \tag{62}$$

Flexure and Direct Load on Rectangular Sections

Compression Failure

The strength of the compression side of the section will be the same when the member is subjected to bending and direct load as it is under flexure alone. Therefore, (60a) can be used to predict the resistance to compression failure of an eccentrically loaded rectangular section.

When there is sufficient tensile steel to prevent a tension failure, the ultimate compressive moment is (see Fig. 12)

$$M_u = P_u e = \frac{1}{3} b d^2 f'_c + (d-d') A'_s f_y$$

from which, since $d = \frac{t+2d''}{2}$

$$P_u = \frac{2 A'_s f_y}{\frac{2e'}{(d-d')} + 1} + \frac{b t f'_c}{\frac{3te'}{d^2} + \frac{6dt - 3t^2}{2d^2}} \tag{63}$$

Eq. (63) gives the theoretical value when the eccentricity of the load is greater than the eccentricity of the resisting forces in the compression side of the section. Because of its derivation it has no theoretical meaning for smaller eccentricities but it can be adjusted to the smaller range by making P approach the proper value for an axially loaded column as e' approaches zero. The first term giving the steel strength needs no adjustment because it equals 2 $A'_s f_y$ when e' equals zero. It is known that the concrete strength is 0.85 btf'_c for axial load so the second term becomes

$$\frac{btf'_c}{\frac{3te'}{d^2} + 1.178}$$

and with this change, Eq. (63) becomes:

$$P_u = \frac{2A'_s f_y}{\frac{2e'}{(d-d')} + 1} + \frac{btf'_c}{\frac{3te'}{d^2} + 1.178} \tag{64}$$

which gives the strength of the rectangular member as controlled by compression strength.

Tensile Failure

If the member is over-reinforced on the compression side so that the compression steel is sufficient to take the total compression force without help from the concrete, the value of the ultimate load can be expressed by taking moments, and

$$P_u = A_s f_y \frac{2(d-d')}{2e' - (d-d')} \tag{65}$$

When there is not sufficient compression steel to take all the compression force, it may be assumed for practical purposes that both the tension and compression steel will be stressed to the yield point at ultimate load, and the remainder of the compression load on the section acts on the compression side of the concrete over an equivalent depth equal to a. This total compression on the concrete is then 0.85 abf'_c and it follows that

$$P_u + A_s f_y = A'_s f_y + 0.85\,abf'_c \quad \text{or,} \quad 0.85\,abf'_c = P_u + (A_s - A'_s) f_y$$

and
$$a = \frac{P_u}{0.85\,bf'_c} + (A_s - A'_s)\frac{m}{b}$$

Then moments about the tension steel

$$M_u = P_u\left(e' + \frac{d-d'}{2}\right) = (d-d')A'_s f_y + 0.85\,f'_c ab\left(\frac{d-d'}{2} + \frac{t}{2} - \frac{a}{2}\right) \tag{66}$$

Substituting the previous values and solving for P_u, we find:

$$P_u = 0.85\,tbf'_c\left\{-\left[\frac{e'}{t} - 0.5 + (p-p')m\right] + \sqrt{\left[\frac{e'}{t} - 0.5 + (p-p')m\right]^2 + \frac{2(d-d')}{t}p'm + (p-p')m\left[\frac{d-d'}{t} + 1 - (p-p')m\right]}\right\} \tag{67a}$$

For the symmetrical reinforcement, when p = p', this reduces to

$$P_u = 0.85\, t b f_c' \left\{ \sqrt{\left(\frac{e'}{t} - 0.5\right)^2 + \frac{d-d'}{t} p_t m} - \left(\frac{e'}{t} - 0.5\right) \right\} \tag{67b}$$

in which, $p_t = p + p' = \dfrac{A_s + A_s'}{bt}$

When there is no compressive steel, p' = 0 and the ultimate load equals

$$P_u = 0.85\, t b f_c' \left\{ \sqrt{\left(\frac{e'}{t} - 0.5 + pm\right)^2 + \left(\frac{d-d'}{t} + 1 - pm\right) pm} - \left(\frac{e'}{t} - 0.5 + pm\right) \right\} \tag{67c}$$

Columns with Round Cores

Square Columns—When in a square column (Fig. 13(a)), the longitudinal reinforcement is distributed around the circumference of a circle with a diameter d, Eqs. (64) and (67) can be adapted by substituting for the value of d' the equivalent effective steel distance in terms of d. Using 0.67d and assuming half the steel effective on each side of the section, Eq. (64) becomes, for ultimate load producing compression failure:

$$P_u = \frac{A_{st} f_y}{\frac{3e'}{d} + 1} + \frac{0.85 A_c f_c'}{\frac{10.2\, t e'}{(t + 0.67d)^2} + 1.51} \tag{68}$$

in which A_{st} = cross-sectional area of bars

Round Columns—To adapt Eq. (68) for compression failure to the case of the round column with round core, the effective depth of the concrete section t, can be changed to 0.8D and Eq. (68) becomes

$$P_u = \frac{A_{st} f_y}{\frac{3e'}{d} + 1} + \frac{0.85 A_c f_c'}{\frac{8.16\, De'}{(0.8D + 0.67d)^2} + 1.51} \tag{69}$$

For the case of tension failure a new equation can be derived in the same manner as Eq. (67). For this purpose the distance from the center line of the section to the center of gravity of the area of concrete in compression can be expressed with sufficient accuracy as

$$\bar{x} = 0.211\, D + 0.293 \left(0.785\, D - \frac{2A}{D}\right) \tag{70}$$

in which A is the compression area. Assuming that the total steel compression is equal to the total steel tension

$$P_u = 0.85 A f_c' \text{ or } A = \frac{P_u}{0.85 f_c'}$$

Then

$$\bar{x} = 0.211D + 0.293\left(0.785D - \frac{2P_u}{0.85Df'_c}\right)$$

Then assuming that four-tenths of the total steel area is effective on each side, and that the effective d-d' is 0.75 d, adding 0.09D to e' to allow for deflection, and taking moments:

$$P_u(e' + 0.09D + 0.375d) = (0.40A_{st} \times 0.75f_y d) + P_u(\bar{x} + 0.375d)$$

which reduces to the following equation for the ultimate load controlled by tension in a round column with a round core:

$$P_u = 0.85D^2 f'_c \left[\sqrt{\left(\frac{0.85e'}{D} - 0.3\right)^2 + \frac{dp_t m}{2.5D}} - \left(\frac{0.85e'}{D} - 0.3\right)\right] \qquad (71)$$

Unsymmetrically Reinforced Rectangular Sections

The maximum axial load capacity of an unsymmetrically reinforced rectangular column is given by:

$$P_o = (A'_s + A_s)f_y + 0.85btf'_c \qquad (72)$$

when the load is concentric with the resultant of the resisting forces in the steel and concrete. The centroid of the resisting forces is located at a distance Δe from the center of the cross-section, where

$$\Delta e' = \frac{A'_s(m-1)\left(\frac{t}{2} - d'\right) - A_s(m-1)\left(d - \frac{t}{2}\right)}{bt + (A'_s + A_s)(m-1)} \qquad (73)$$

where $A_s > A'_s$

The complete moment thrust curve for rectangular columns with unsymmetrical reinforcement may be drawn as follows:

Given, e' the eccentricity from the center of the cross-section, positive on the side with the smaller steel area, construct the curve for positive eccentricity using Eq. (64) and Eq. (67a).

Plot $(P_o, -P_o \Delta e')$ as given by Eq. (72) and (73). Draw a straight line from this point to P_u at e' = o from Eq. (64) to obtain Curve A.

Curve B, the compression failure curve for negative eccentricity corresponding to Eq. (64) for positive eccentricity, may be obtained by joining $(P_o, -P_o \Delta e')$ to the point P = o, $P_u e' = -(A_s f_y (d-d') + \frac{1}{3} f'_c bd^2)$ with a straight line.

(Curve C is obtained by applying Eq. (64) also when the load is on the side of the greater steel, and is shown for comparison only.)

The tension failure curve for negative values of e' is given by Equations (74), (75a) and (75b).

For $\frac{a}{2} \geq d'$

$$P_u\left(-e' - \frac{t}{2} + d'\right) = A_s f_y (d - d') + (P_u + A'_s f_y - A_s f_y)\left(d - \frac{a}{2}\right) \tag{74}$$

where

$$a = \frac{P_u + A'_s f_y - A_s f_y}{0.85 f'_c b}$$

For $\frac{a}{2} = d'$

$$P_u\left(-e' - \frac{t}{2} + d'\right) = A'_s f_y (d - d') \tag{75a}$$

It may be noted that Curve (75a) is a straight line. For $\frac{a}{2} \leq d'$

$$P_u\left(-e' - \frac{t}{2} + \frac{a}{2}\right) = A'_s f_y \left(d - \frac{a}{2}\right) \tag{75b}$$

where

$$a = \frac{P_u + A'_s f_y}{0.85 f'_c b}$$

Curves (74) and (75b) join Curve (75a) when $\frac{a}{2} = d'$.

A typical moment thrust curve for an unsymmetrically reinforced rectangular column is plotted in Fig. 14.

Substitution of numerical values in the previously derived equations reveals that the essential difference between the various stress distributions lies in the maximum percentage of reinforcement, p_b, (called balanced reinforcement) which will fully develop the compressive strength of the beam. Tables 5a and 5b give values of balanced reinforcement and the corresponding ultimate capacity for various assumptions regarding the value of E_c and ultimate strain. Table 5a shows that when f_y is less than 60,000 psi, the value of $p = 0.40 f'_c/fy$ marked "allowed" is, in most cases, less than that obtained by the various methods. Hence, Tables 5a and 5b show that the equation (3) which limits p to $0.40\ f'_c/f_y$ is satisfactory. In this study it should be noted that conservative values of $\epsilon_u = 0.003$, 0.0035 and 0.0038 have been used. The value of $\epsilon_u = 0.003$ represents the smallest maximum strain which might be expected. Strains obtained in tests are generally in excess of 0.003.

Fig. 15a to Fig. 16, inclusive, illustrate the effect of variations in the assumed value of E_c and ϵ_u for members subject to combined bending and axial load. It is to be noted that changes in the value of ϵ_u have negligible effects on the moment-thrust curves.

In Fig. 17, ultimate strength curves as computed by three different methods have been plotted. It should be noted that the parabolic distribution gives somewhat more conservative values than the trapezoidal distribution. The values obtained by rectangular stress distribution as computed by equation (13) plot between the values given by the assumed trapezoidal and parabolic distributions when the ultimate capacity is controlled by compression. The lower portion of the moment-thrust curve for the rectangular and trapezoidal distributions coincide.

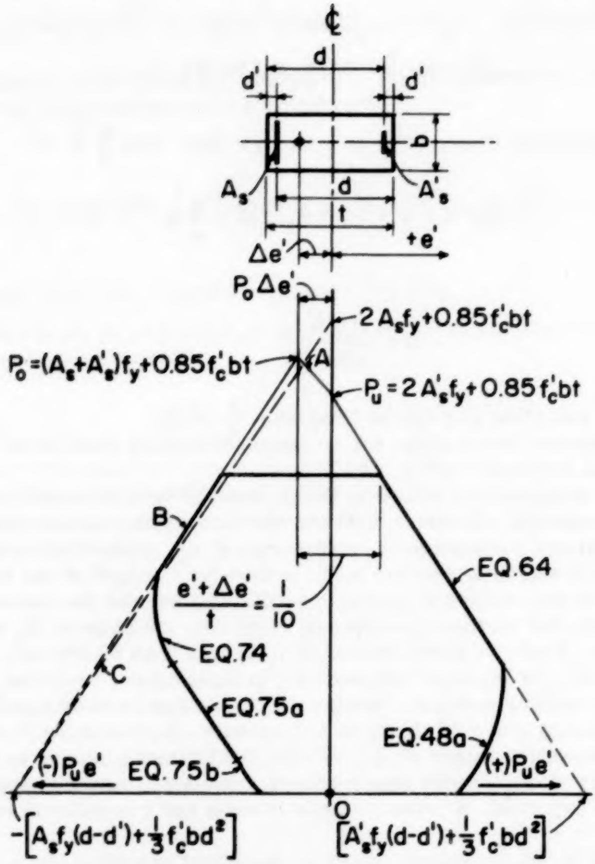

Fig. 14. Graphical Solution for an Unsymmetrically Reinforced Member Subject to Combined Bending and Axial Load.

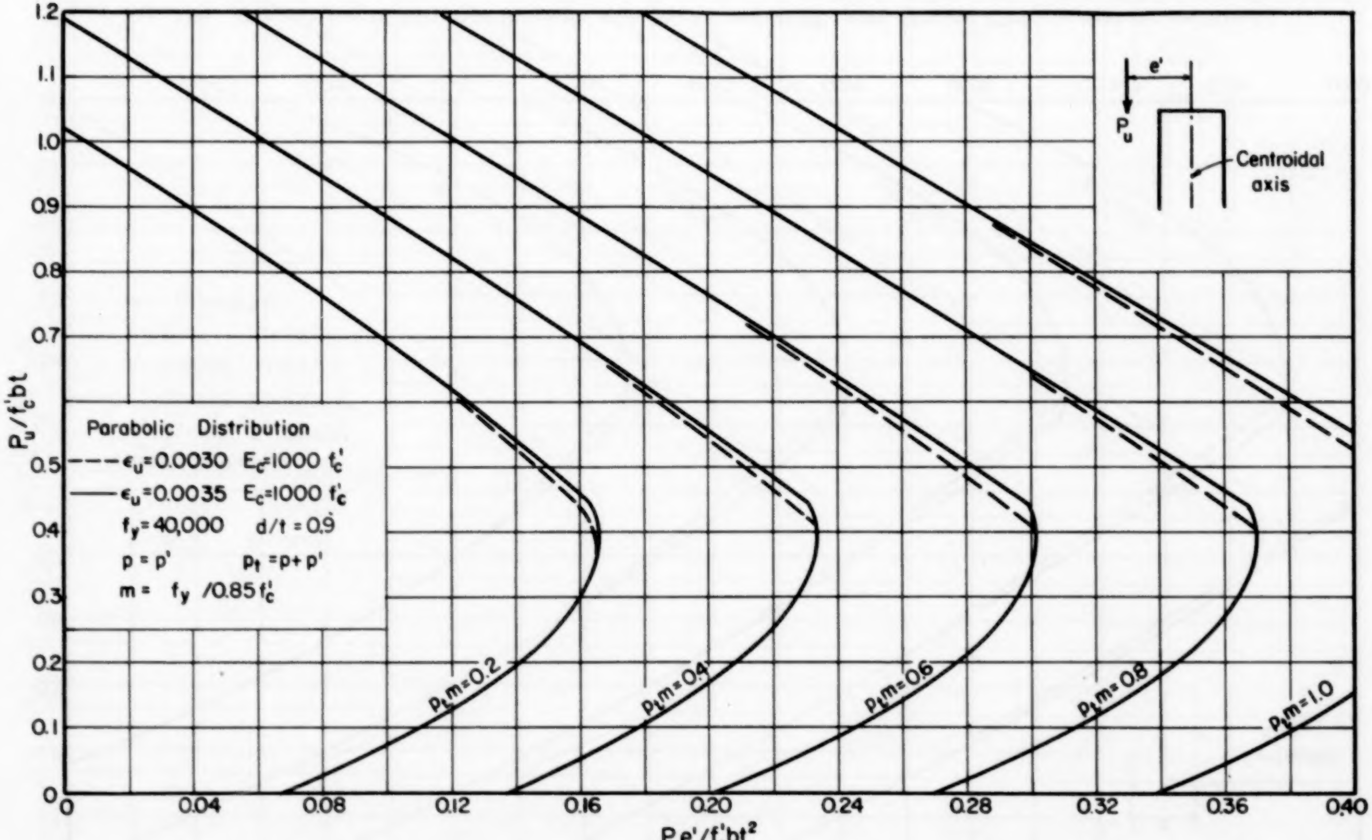

Fig. 15a. Comparison of Ultimate Strength with Variations in Ultimate Strains (Parabolic Stress Block).

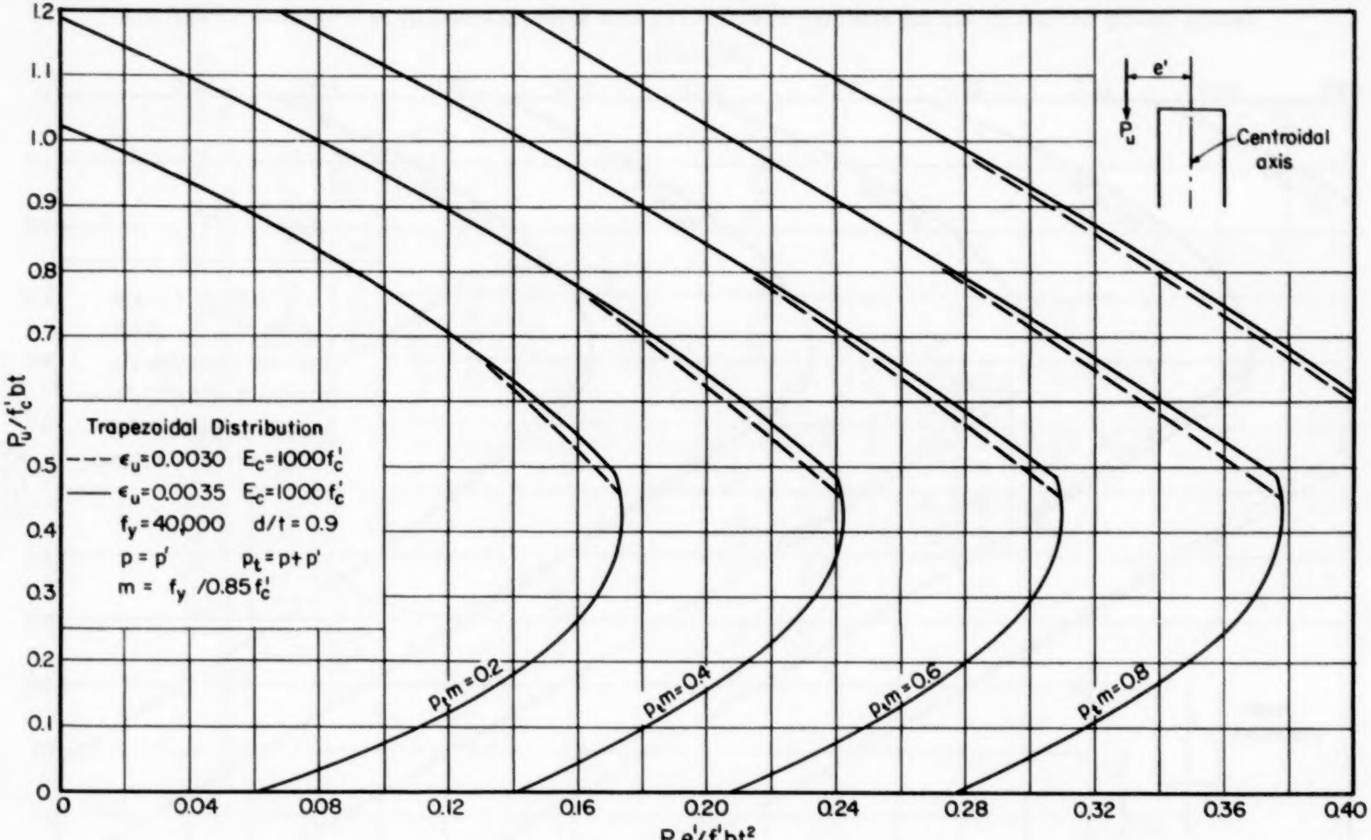

Fig. 15b. Comparison of Ultimate Strength with Variations in Ultimate Strains (Trapezoidal Stress Block).

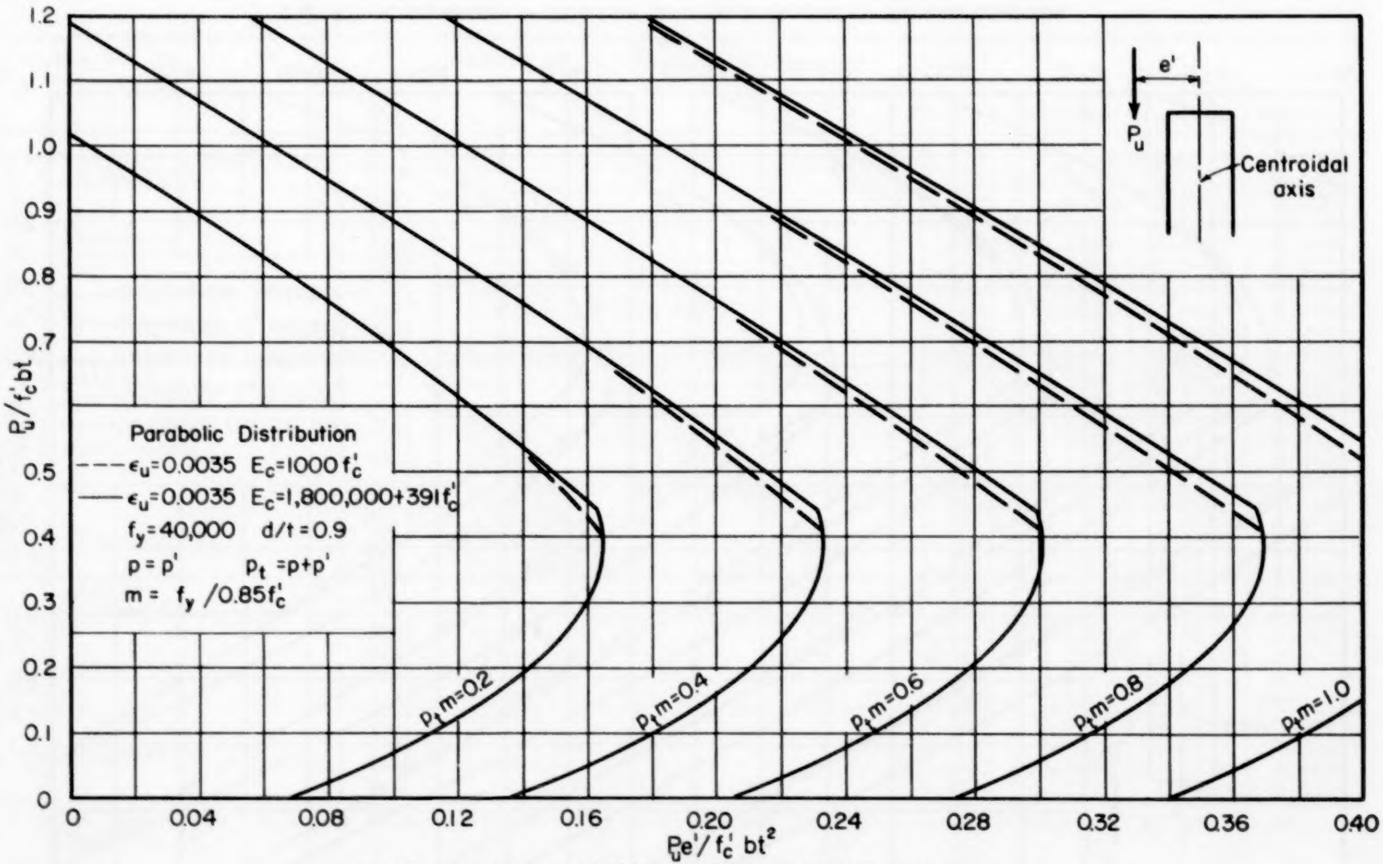

Fig. 16. Comparison of Ultimate Strength with Variations in E_c.

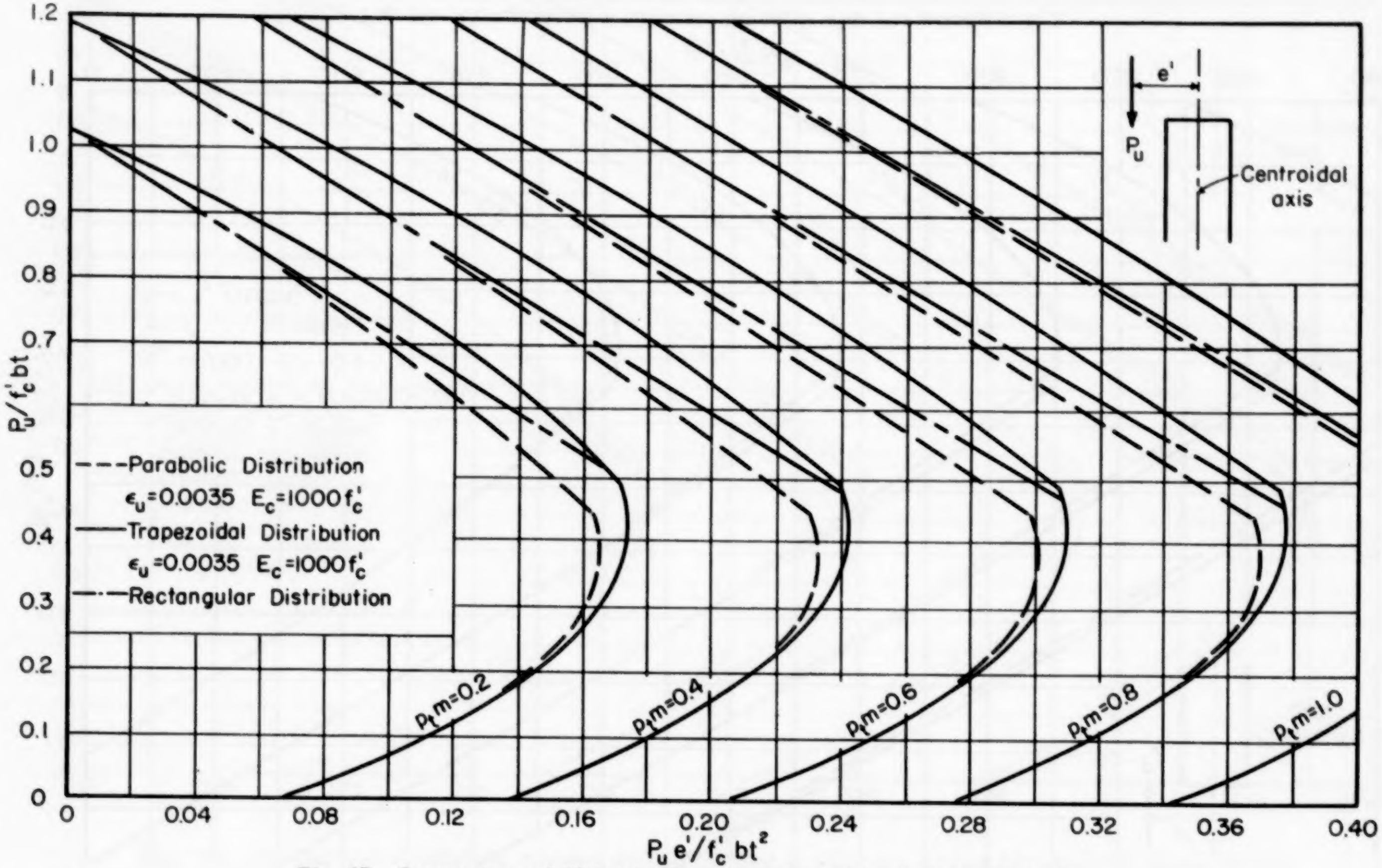

Fig. 17. Comparison of Ultimate Strength Indicated by Various Methods.

Table No. 5a — Values of Balanced Reinforcement

f_y		ϵ_u	E_c	$f'_c=3000$	$f'_c=4000$	$f'_c=5000$	$f'_c=6000$
f_y = 40,000	Allowed: $p=0.40^*f'_c/f_y$	.		0.0300	0.0400	0.0500	0.0562
	Rectangular			0.0342	0.0456	0.0570	0.0684
	Parabolic	.0035	1000 f'_c	0.0369	0.0492	0.0615	0.0738
		.0035	1,800,000 + 391 f'_c	0.0368	0.0477	0.0584	0.0686
		.0030	1000 f'_c	0.0344	0.0459	0.0573	0.0689
		.0038	1,800,000 + 391 f'_c	0.0381	0.0496	0.0607	0.0716
	Trapezoidal	.0035	1000 f'_c	0.0404	0.0540	0.0675	0.0810
		.0035	1,800,000 + 391 f'_c	0.0404	0.0526	0.0645	0.0760
		.0030	1000 f'_c	0.0379	0.0505	0.0631	0.0758
		Var.	6,000,000 $f'_c/(f'_c + 2000)$	0.0390	0.0450	0.0512	0.0580
f_y = 50,000	Allowed			0.0240	0.0320	0.0400	0.0450
	Rectangular			0.0274	0.0365	0.0456	0.0547
	Parabolic	.0035	1000 f'_c	0.0277	0.0369	0.0461	0.0553
		.0035	1,800,000 + 391 f'_c	0.0276	0.0358	0.0437	0.0513
		.0030	1000 f'_c	0.0255	0.0341	0.0426	0.0511
		.0038	1,800,000 + 391 f'_c	0.0287	0.0373	0.0456	0.0537
	Trapezoidal	.0035	1000 f'_c	0.0304	0.0405	0.0506	0.0607
		.0035	1,800,000 + 391 f'_c	0.0303	0.0395	0.0483	0.0569
		.0030	1000 f'_c	0.0281	0.0376	0.0469	0.0563
		Var.	6,000,000 $f'_c/(f'_c + 2000)$	0.0286	0.0327	0.0371	0.0421
f_y = 60,000	Allowed			0.0200	0.0267	0.0333	0.0375
	Rectangular			0.0228	0.0304	0.0380	0.0456
	Parabolic	.0035	1000 f'_c	0.0217	0.0289	0.0361	0.0433
		.0035	1,800,000 + 391 f'_c	0.0217	0.0280	0.0342	0.0402
		.0030	1000 f'_c	0.0196	0.0260	0.0325	0.0390
		.0038	1,800,000 + 391 f'_c	0.0225	0.0293	0.0358	0.0422
	Trapezoidal	.0035	1000 f'_c	0.0238	0.0317	0.0396	0.0475
		.0035	1,800,000 + 391 f'_c	0.0238	0.0309	0.0378	0.0445
		.0030	1000 f'_c	0.0215	0.0287	0.0356	0.0430
		Var.	6,000,000 $f'_c/(f'_c + 2000)$	0.0220	0.0250	0.0283	0.0321

*The coefficient 0.40 is reduced at the rate of 0.025 per 1000 psi concrete strength in excess of 5000 psi.

Table No. 5b — Maximum Capacity in Terms of $M_u/f'_c bd^2$

		ϵ_u	E_c	$f'_c=3000$	$f'_c=4000$	$f'_c=5000$	$f'_c=6000$
$f_y = 40,000$	Allowed			0.306	0.306	0.306	0.292
	Rectangular			0.333	0.333	0.333	0.333
	Parabolic	.0035	$1000\ f'_c$	0.336	0.336	0.336	0.336
		.0035	$1,800,000 + 391\ f'_c$	0.335	0.329	0.326	0.321
		.0030	$1000\ f'_c$	0.322	0.322	0.322	0.322
		.0038	$1,800,000 + 391\ f'_c$	0.339	0.337	0.333	0.330
	Trapezoidal	.0035	$1000\ f'_c$	0.367	0.367	0.367	0.367
		.0035	$1,800,000 + 391\ f'_c$	0.367	0.361	0.357	0.353
		.0030	$1000\ f'_c$	0.353	0.353	0.353	0.353
		Var.	$6,000,000\ f'_c/(f'_c + 2000)$	0.382	0.345	0.321	0.305
$f_y = 50,000$	Allowed			0.306	0.306	0.306	0.292
	Rectangular			0.333	0.333	0.333	0.333
	Parabolic	.0035	$1000\ f'_c$	0.323	0.323	0.323	0.323
		.0035	$1,800,000 + 391\ f'_c$	0.323	0.318	0.313	0.309
		.0030	$1000\ f'_c$	0.309	0.309	0.309	0.309
		.0038	$1,800,000 + 391\ f'_c$	0.331	0.326	0.321	0.318
	Trapezoidal	.0035	$1000\ f'_c$	0.354	0.354	0.354	0.354
		.0035	$1,800,000 + 391\ f'_c$	0.354	0.349	0.344	0.339
		.0030	$1000\ f'_c$	0.338	0.338	0.338	0.338
		Var.	$6,000,000\ f'_c/(f'_c + 2000)$	0.362	0.322	0.298	0.284
$f_y = 60,000$	Allowed			0.306	0.306	0.306	0.292
	Rectangular			0.333	0.333	0.333	0.333
	Parabolic	.0035	$1000\ f'_c$	0.313	0.313	0.313	0.313
		.0035	$1,800,000 + 391\ f'_c$	0.313	0.306	0.301	0.297
		.0030	$1000\ f'_c$	0.292	0.292	0.292	0.292
		.0038	$1,800,000 + 391\ f'_c$	0.319	0.315	0.310	0.307
	Trapezoidal	.0035	$1000\ f'_c$	0.342	0.342	0.342	0.342
		.0035	$1,800,000 + 391\ f'_c$	0.342	0.336	0.331	0.326
		.0030	$1000\ f'_c$	0.320	0.320	0.320	0.320
		Var.	$6,000,000\ f'_c/(f'_c + 2000)$	0.341	0.302	0.278	0.265

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c. Discussion of several papers, grouped by Divisions.